DOI: 10.1478/C1C8902002

AAPP | Atti della Accademia Peloritana dei Pericolanti
Classe di Scienze Fisiche, Matematiche e Naturali
ISSN 1825-1242

Vol. 89, No. 2, C1C8902002 (2011)

BEAUTY OF MATHEMATICS

BORIS MORDUKHOVICH*

(Communication presented by Prof. Carmela Vitanza)

ABSTRACT. The paper reflects the author's personal view on the essence of Mathematics and its impact to the modern society.

It is probably hard to find a person who does not value the crucial impact of the Sciences in the technological and cultural progress of the society. And Mathematics is of course the Queen of the Sciences! But it may happen that in thinking and talking about Mathematics, some people emphasize its merely educational component.

There is no doubt that Mathematics is the basis of Science, Engineering and Technology. It is not easy to find a modern educated person who does not have a solid knowledge of Mathematics and is not able to use it in his or her everyday work and life. Thus to teach Mathematics at all levels of the educational system is an extremely important and honorable part of mathematical activity that is difficult to overstate.

However, besides the educational importance of Mathematics, I would like to emphasize that Mathematics has its own research component, and there are a great many of unsolved problems in Mathematics and its applications, which makes this discipline very attractive for researchers.

Mathematics is definitely a Science, but of a special type in comparison with all other Sciences. As a Science, Mathematics is based on logic and it is often, but not always, oriented towards applications. On the other hand, among the strongest criteria in Mathematics (in both its pure and applied areas) are beauty and harmony. The most powerful results in Mathematics are surely the most beautiful, and the esthetic component is a driving force of many mathematical proofs. From this viewpoint, Mathematics is close to Fine Arts.

In Mathematics, we do not like to label particular fields of our research as"pure" and "applied". Mathematics is fully unified; it is not intrinsically divided into Pure and Applied Mathematics, as some people, usually non-mathematicians, often do. The same laws and principles rule all the areas in Mathematics, and we may only conditionally label say, Algebra as a part of Pure Mathematics and Partial Differential Equations as an area of Applied Mathematics.

Lecture delivered by the author on the occasion of the awarding of the *Laurea Magistrale Honoris Causa* in Mathematics by the Università degli Studi di Messina (Messina, 23 June 2011).

I am a mathematician not only on the formal basis of my education and current occupation, but also (and mainly) due to my way of thinking, understanding, and acting. During many years of my mathematical career I have been involved in active work in both "pure" and "applied" areas of Mathematics as well as in practical applications of mathematical modeling and results to, *e.g.*, engineering, economic, military and environmental problems. I have never felt any essential difference between the way how I dealt with "pure" and "applied" problems. I just did Mathematics.

The main area of my current research interests is Optimization and Variational Analysis [1]. This is a very important and active part of the Mathematical Sciences, with a strongly developed and still challenging mathematical theory and numerous applications to Economics, Engineering, Applied Sciences, and many other areas of human activity. This field of Mathematics provides an excellent example of how difficult and in fact it is senseless to split Mathematics into "pure" and "applied" areas.

Linguistically the word "optimization" means finding the best solutions. By Optimization mathematicians mean theoretical results, methods, and algorithms that allow us first to establish the existence of best solutions in mathematical models as maxima or minima of some functions subject to various constraints, then to single out them by appropriate mathematical conditions, and finally to develop efficient computational techniques to find them numerically. Optimization theory deals not only with problems on maxima and minima, but also with more complicated situations when we look for various equilibria that are particularly important in Economics and other fields of Applied Sciences.

Optimization ideas and so-called variational principles play a crucial role not only to solve problems on maxima, minima and equilibria, but also in many areas of Mathematics and applications that are not of optimization nature.

It was nicely and deeply said in 1744 by Leonhard Euler, who was one of the best mathematicians of all time: "Namely, because the shape of the whole universe is most perfect and, in fact, designed by the wisest creator, nothing in all of the world will occur in which no maximum or minimum rule is somehow shining forth." (In Latin: *Quum enim mundi universi fabrica sit perfectissima, atque a Creatore sapientissimo absoluta, nihil omnino in mundo contingint, in quo non maximi minimive ratio quapiam eluceat.*) [2].

In fact, optimization principles and techniques have played a fundamental role in Mathematics and its applications from the beginning of mathematical analysis. In particular, the concept of the derivative, which is now among the most important and useful mathematical constructions, was introduced by the French mathematician Pierre de Fermat in 1636 for the purpose of solving an optimization problem [3]. By the way, Fermat was a lawyer in his main profession and did Mathematics just for pleasure, as a hobby. In the non-mathematical community he is mostly famous for his "Fermat's Last Theorem", actually a conjecture, from Number Theory, which was very simply formulated and thus attracted the attention of many mathematicians and non-mathematicians, but was solved quite recently by advanced methods of modern mathematics that were surely not known to Fermat.

Modern optimization theory and variational principles are much different from those developed in the time of Fermat and Euler. They are very powerful but require advanced mathematical tools, such as generalized derivatives. But based on these principles and techniques, we can solve many challenging mathematical and applied problems that are important nowadays.

It is my pleasure to emphasize great contributions of many Italian mathematicians to Optimization and Equilibrium Theories, Calculus of Variations and Optimal Control, and other areas of Variational Analysis and its Applications. In particular, Joseph-Louis Lagrange (born Giuseppe Luigi Lagrangia in Torino) was one of the greatest mathematicians of all time, a founder of Optimization Theory and of Mathematical Analysis in general; Giuseppe Peano discovered in the end of the 19^{th} century and at the beginning of the 20^{th} century several fundamental mathematical concepts that play crucial roles in modern Variational Analysis; Leonida Tonelli was the main contributor to the Calculus of Variations in the $20s$ century, and on and on. Nowadays Italy is undoubtedly among the world leaders in Variational Analysis, Optimization, and related areas.

But Mathematics does have any boundaries!

References

[1] B. S. Mordukhovich, *Variational Analysis and Generalized Differentiation, I: Basic Theory; II: Applications*, in Grundlehren Series (Fundamental Principles of Mathematical Science), Vols. 330 and 331 (Springer-Verlag, Berlin, Germany, 2006).

[2] L. Euler, *Methodus Inveniendi Curvad Lineas Maximi Minimive Proprietate Gaudentes Sive Solution Problematis Viso Isoperimetrici Latissimo Sensu Accepti* (Lausanne, Switzerland, 1744); reprinted in *Opera Omnia*, Ser. 1, Vol. 24 (1952).

[3] P. de Fermat, *Letters to M. Mertsenne and J. Roberval* (Toulouse, France, 1676).

---

* Wayne State University
Detroit (USA)

Email: boris@math.wayne.edu

Presented 30 November 2011; published online 1 December 2011