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ON SPECIAL- AND GENERAL-RELATIVISTIC THERMODYNAMICS

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ABSTRACT. In reviewing approaches to the special- and general-relativistic theory of irreversible thermodynamical processes near equilibrium, problems of this procedure and possible solutions are discussed.

1. Introduction

Continuum thermodynamics is concerned with the general structure of sufficiently small Schottky systems, each of which denoted by position and time, exchanging heat, work and material with its adjacent Schottky systems. (For a survey on different formulations of non-relativistic continuum thermodynamics, see [1].) In the present contribution, we ask what happens when position and time mark points in a Minkowski or curved Riemann space-time. In other words, we ask for the relation of continuum thermodynamics to the theories of special and general relativity.

Our approach is pragmatic, insofar as we assume that thermodynamics and relativity theory in their classical phenomenological versions have a common field of application in relativistic astrophysics and cosmology for which one should try to find a quasi-axiomatic relativistic continuum thermodynamics. According to this approach, one starts from balance and constitutive equations, without demanding an action integral and/or a microscopic model for their justification. One has only to require that the basic (quasi-axiomatic) laws of such a theory are adopted to existing material classes describable by a Lagrangian and/or microscopic models. (For a discussion of this standpoint, see [2].)

Relativists often prefer a "more fundamental" approach based on a variational principle and relativistic microscopic (kinetic and statistical) models. Of course, it would be desirable if one had such a theory. Possibly, problems discussed below have their origin in the deficiency of the quasi-axiomatic approach. But, otherwise, most work done in general relativity assumes this pragmatic standpoint, too. Why not to attempt the same for relativistic thermodynamics? If there arise problems it is more probable that they are due to the fact that one always meets with obstacles when one tries to unify an irreversible with a reversible theory. This is problematic on all levels, also on the level of microscopic models. And, as far as the desire to have a variational principle is concerned, the problems one encounters in the approach chosen here became even tenser when one would try to start from it. For, the Hamiltonian principle is a suitable tool in reversible, but not in irreversible physics (for this problem, see [3, 4, 5]). An additional ground for difficulties in unifying

thermodynamics and general relativity is that notions like energy being fundamental for thermodynamics generally cannot be defined in general relativity [6].

In the present paper, we shall comment on the situation in relativistic continuum thermodynamics near equilibrium because it is best elaborated[1]. Its non-relativistic formulation is called "Thermodynamics of Irreversible Processes (TIP)" such that, in the following, we shall speak of relativistic TIP. In this context, the discussion will be limited to simple fluids to show the problems one meets with. Incorporation of additional wanted fields like an intrinsic spin density would not change the character of the arising problems, but make them only more critical.

2. Relativistic theory of irreversible thermodynamical processes near equilibrium

2.1. Basic relations. Relativistic TIP (see, e.g., [2, 7, 11, 12]) is based on the assumption that an arbitrary local state of the fluid is specified by the primary variables: particle flow vector N^i, energy-momentum tensor of fluid matter ${}_mT^{ik}$, spin density S^{ikl}, and entropy flux vector S^i. Relativistic TIP postulates that these quantities satisfy the following relations:[2]

(1) **Conservation of mass (or particle number):** $N^k{}_{;k} = 0,$

(2) **balance of energy-momentum:** ${}_mT^{ik}{}_{;k} = F^i,$

(3) **balance of moment of spin density:** $S^{ikl}{}_{;l} = \frac{1}{c}x^{[i}F^{k]} + L^{ik},$

(4) **second law of thermodynamics:** $S^k{}_{;k} \geq 0.$

Here, F^i is a 4-force density and $L^{ik} = -L^{ki}$ is a couple tensor. The semicolon denotes the covariant derivative in a Riemann space-time (or, if one thinks of generalizations of general relativity, in a more general curved space-time) which, in the Minkowski space-time, reduces to the partial derivative. The first three relations are balance equations which, in the case of special relativity can be reformulated into integral balance equations that, in the special cases of vanishing supply terms, $F^i = 0$, $L^{ik} = 0$, represent conservation laws. The latter relation expresses the positive entropy production.

Before turning to critical comments and possible modifications, let us discuss the three balance relations (1, 2, 3) in some more detail:

ad (1): In the framework of the theory of special relativity, it was shown by Eckart [7] that the particle flow vector N^i has to be introduced independently of the energy-momentum tensor T^{ik}. Eckart's choice was

(5) $$N^i = \rho u^i$$

(ρ denotes the proper particle density and u^i the 4-velocity of the fluid). In special relativity, this vector satisfies in Gaussian coordinates the relation $N^k{}_{,k} = 0$ and, in general relativity, its generally covariant version (1). It states the conservation of the number of

[1] In its special relativistic version, it was formulated by Eckart [7] for simple fluids and in five papers by Kluitenberg et al [8, 9, 10] extended to physically much more realistic matter configurations.

[2] We follow [2] since therein an external input given by supply quantities F^i, L^{ik} and the balance of the spin density S^{ikl} are taken into consideration; later the condition will be considered under which such terms are admissible in general relativity.

particles in the region B given by $\int_B \rho d^3x$. Therefore, matter has to be interpreted as particle number and not as inertia. This is an extension of the non-relativistic approach (found in two foregoing papers by Eckart [13] and, later, in papers by Meixner [14] and de Groot and Mazur [12]) to relativity theory.[3]

ad (2): Furthermore, following Eckart [7], for a symmetric energy-momentum tensor and vanishing spin and supply terms, relativists argue that the first law of thermodynamics is closely related to the conservation of energy-momentum which in its general-covariant form is given by eq. (2). To show this relationship, the energy-momentum tensor satisfying the relation,

$$ {}_mT^{ik}{}_{;k} = 0, \tag{6} $$

is represented as (with $u^iu_i = -c^2$, $h^i{}_k = \delta^i{}_k + c^{-2}u^iu_k$)

$$ {}_mT^{ik} = \frac{1}{c^2}(eu^iu^k + q^iu^k + u^iq^k) + ph^{ik} + \pi^{ik} \tag{7} $$

where the energy density e, the heat flow q^i, the isotropic pressure p, and the anisotropic pressure π^{ik} are defined as follows (with ρ =particle density and u =internal energy):

$$ e = \rho c^2 + u \;:=\; c^{-2}{}_mT^{ik}u_iu_k, \tag{8} $$

$$ q^i \;:=\; -{}_mT^{ln}u_nh^i{}_l, \tag{9} $$

$$ \pi^{ik} = \pi^{ki} \;:=\; {}_mT^{ln}h^i{}_lh^k{}_n - \frac{1}{3}ph^{ik}, \quad p := \frac{1}{3}{}_mT^{ik}h_{ik}. \tag{10} $$

Then, the relativistic first law can be identified as the scalar equation following by multiplication of eq. (6) by u_i and rewriting it, by use of eq. (1) and the splitting $e = \rho c^2 + u$ given in (8), in an equation for the specific internal energy $\underline{u} := \rho^{-1}u$,

$$ \rho\dot{\underline{u}} + p\Theta + q^k{}_{;k} + \frac{1}{c^2}\dot{u}_kq^k + \sigma_{ik}\pi^{ik} = 0. \tag{11} $$

The kinematic invariants shear σ_{ik}, rotation ω_{ik}, expansion Θ, and acceleration $\dot{u}_i$ appearing in eq. (11) are given by the expressions,

$$ \sigma_{in} \;:=\; u_{(i;n)} + \dot{u}_{(i}u_{n)} - \frac{\Theta}{3}h_{in} \tag{12} $$

$$ \omega_{in} \;:=\; u_{[i;n]} + \dot{u}_{[i}u_{n]} \tag{13} $$

$$ \Theta \;:=\; u^i{}_{;i} \tag{14} $$

$$ \dot{u}_i \;:=\; u_{i;n}u^n. \tag{15} $$

In deriving the relativistic relation (11), again, non-relativistic continuum thermodynamics is the guideline. Also there, the balance for the internal energy, which is deducable from the balance for the total energy, is assumed to be the equivalent of the first law of thermodynamics [1].

For rewriting the energy balance contained in eq. (2) into the balance of the specific internal energy one had to assume the balance (1) with the choice (5), where the latter relation says that there is only a convective mass (or particle) flow. Thus, from the thermodynamical point of view, the introduction of (1), independently of (2), has a well-determined function. It enables one to deduce from (2) the first law of (continuum) thermodynamics.

[3] For the link between energy-momentum and mass conservation in a Lagrangian framework, see [15].

This approach is suggested by non-relativistic continuum thermodynamics. But, in the case here under consideration, it is insofar confusing as eq. (1) interpreted as mass conservation is no relativistic relation of its own right because all aspects of mass-energy conservation should be embraced by relation (2).

Therefore, it should be useful to have a closer look at the relation between the balances (1) and (2) under the aspect of the compatibility of their relativistic corrections (for a detailed discussion, see [16]). In the first-order approximation $N^k{}_{,k} = 0$ and ${}_mT^{ik}{}_{,k}u_i = 0$ lead to the same result, namely the Newtonian mass-balance law

$$\frac{\partial \rho}{\partial t} + v^\nu \rho_{,\nu} + \rho v^\nu{}_{,\nu} = 0. \tag{16}$$

Here v^μ denotes the velocity of the fluid elements. In this order of approximation the condition (1) seems to be unnecessary. As expected, the mass conservation is a non-relativistic implication of eq. (2). However, the second-order approximation of $N^k{}_{,k} = 0$ gives

$$0 = \frac{1}{2c^2}\Big[\frac{v^2}{2}\left(\frac{\partial \rho}{\partial t} + v^\nu \rho_{,\nu} + \rho v^\nu{}_{,\nu}\right) + \rho(v_\mu v_\nu v^{\mu,\nu} + v_\nu \dot{v}^\nu)\Big], \tag{17}$$

while the second-order approximation of ${}_mT^{ik}{}_{,k}u_i = 0$ becomes

$$\begin{aligned} 0 = \frac{v^2}{2}\left(\frac{\partial \rho}{\partial t} + v^\nu \rho_{,\nu} + \rho v^\nu{}_{,\nu}\right) + \frac{\partial u}{\partial t} + v^\nu u_{,\nu} + (u+p)v^\nu{}_{,\nu} + \\ +\rho(v_\nu \dot{v}^\nu + v_\mu v_\sigma v^{\mu,\sigma}) + q^\mu{}_{,\mu} + \pi_{\mu\nu}(v^{\nu,\mu} + v^{\mu,\nu}). \end{aligned} \tag{18}$$

Only the conditions (16) **and** (17) stemming from eq. (1) ensure that (18) becomes the classical law for the internal energy, i.e. the first law of thermodynamics

$$0 = \frac{\partial u}{\partial t} + v^\nu u_{,\nu} + (u+p)v^\nu{}_{,\nu} + q^\mu{}_{,\mu} + \pi_{\mu\nu}(v^{\nu,\mu} + v^{\mu,\nu}). \tag{19}$$

Therefore, viewing eq. (1) as a pure condition for the particle number, by assuming $N^a = nu^a$, where n is the particle number per volume, one cannot reproduce the first law of thermodynamics. However, as was already seen in the above consideration leading to eq. (11), the choice (5) leads to the thermodynamically desired result. But one has to maintain that beside the relativistic energy-momentum conservation law (2) (with $F^i = 0$) there exists a second relativistic mass conservation law (1), which is not beyond all doubt from the relativistic view. For further justification one has to consider higher-order approximation steps of (1) and (2) (this will be done in [16]).

ad (3): Generally, the tensor S^{ikl} in eq. (3) characterizes the total spin containing its orbital and intrinsic parts. In the case $S^{ikl} = 0$, eq. (3) reduces to

$$T^{[ik]} + L^{ik} = 0. \tag{20}$$

Because in general relativity only symmetric energy-momentum tensors satisfying eq. (6) are admissible as source of gravitation, we assumed the symmetric matter tensor (7). If we would assume an asymmetric matter tensor we had to look for a supply term satisfying eq. (20) (see below).

The covariantly formulated continuum thermodynamics given by eqs. (1,2,3,4) describes the behavior of a continuous medium in interaction with an external gravitational

field, i.e., a given gravitational background field which is not assumed to be either a solution of the gravitational equations or generated by the matter under consideration. Hitherto, we did not presuppose any gravitational field equations determining the metric (or, in a more general case, possible other geometric quantities) specifying the space-time curved by gravitation. If one incorporates Einstein's equations (or, in a more general case, the suitably modified field equations), at least, one has to ensure the compatibility of the resulting complete system of equations. Beside the problems described in ad (2) induced by the relativistic formulation, here one meets with the even greater problem of the non-existence of the notion of energy in general relativity, see [6]. Of course, in addition to this, the question as to the thermodynamical nature of the gravitational equations has to be answered. This is of particular interest in the context of the constitutive equations requiring a specification of state spaces. In this context the question if there appear general relativistic degrees of freedom in thermodynamics arises and how to formulate them. Does the curvature or parts of it play this role? Several attempts were made to answer this question, as can be found in black-hole thermodynamics (see, e.g. for an overview [17]), Penrose's Weyl curvature conjecture [18] or the approach assuming a vanishing divergence of the Weyl tensor [19]. Here, this question will not be considered because we confine ourselves on the discussion of material-independent conditions.

2.2. Equilibrium states. Now, we restrict our consideration to a symmetric energy-momentum tensor and vanishing spin and supply terms. That means, we assume eqs. (6) and (7).

In literature, in dependence on the ansatz for the entropy vector, one finds different characterizations of the equilibrium states. In this section, we follow the approach given in [9], and turn to other versions later on. Accordingly, we shall work with the following ansatz for the entropy vector:

$$S^k = -\Theta_i T^{ik} - \phi N^k, \tag{21}$$

where

$$\Theta_i := \frac{u_i}{T} \quad \text{inverse-temperature vector, } T > 0 \text{ temperature} \tag{22}$$

$$\phi := \frac{e - Ts}{\rho T} \quad \text{specific free energy divided by } T \tag{23}$$

$$(s := -c^{-2} u_k S^k, \quad \rho := -c^{-2} u_k N^k).$$

(In [3], the ansatz (21) is justified by the argument that it can be interpreted as the relativistic form of the Carnot-Clausius relation.) Additionally, it is assumed that $N^i = \rho u^i$ (Eckart's assumption), and the particle density is identified with the mass density in the energy-momentum tensor of incoherent matter, ${}_I T^{ik} = \rho u^i u^k$.

Now, equilibrium states can be characterized by the following conditions (equilibrium quantities are marked by $\wedge$):
(i) The entropy 4-vector satisfies the relation saying that the entropy production vanishes (basic condition),

$$\hat{S}^k{}_{;k} = 0. \tag{24}$$

Furthermore, the following conditions are imposed on

$$\Theta^k = \hat{\Theta}^k, \quad \phi = \hat{\phi}, \quad N^k = \hat{N}^k. \tag{25}$$

The latter condition states that the mass flow is no irreversible effect.
(ii) There are no friction processes (no viscosious terms in the energy-momentum tensor) and the fluid is not deformed (mechanical equilibrium conditions),

$$\sigma^{ik} = 0, \quad \Theta = 0, \quad \pi^{ik} = 0. \tag{26}$$

iii) In absence of external fields other than gravity all conductive fluxes vanish (thermal equilibrium conditions),

$$s^l = \frac{q^l}{T} = 0, \tag{27}$$

$$\Psi^l : = T_{,m} h^{ml} + \frac{T}{c^2} \dot{u}^l = \frac{1}{c^2} \left((T u^l)_{;i} - (T u_i)^{;l} \right) u^i = 0, \tag{28}$$

where

$$q^l := -u_k T^{ki} h_i{}^l, \quad s^l := \Theta_k T^{ki} h_i{}^l. \tag{29}$$

As a consequence of these conditions, it follows that, in equilibrium, the medium is a perfect fluid, the relation (24) is equivalent to the Gibbs relation, and the inverse-temperature vector is a Killing vector.

3. Comments on the above-sketched approach

The problems arising in the above-sketched theory are mainly related to (real or supposed) ambiguities, first, in the rules that determine the relativistic equivalents for the non-relativistic thermodynamical quantities "heat" and "entropy" and, second, in the conditions to impose on the equilibrium state. Furthermore, it is somehow unsatisfying that for some balance equations it seems to be no room left in the relativistic framework. To make these points more evident, let us consider the following aspects of the theory here under consideration.

3.1. Heat, energy, and the first law of thermodynamics. In classical thermodynamics, the first law states $\Delta E = Q - W$, where ΔE is the increase in energy content and Q and W are respectively the heat flow into the system from the surroundings and the work done by the system on the surroundings. Following Tolman [20], the transition to its relativistic generalization has to be aware of two aspects of this classical law: It is to be regarded in the first place as expressing the conservation of energy by equating the total energy change in the system to that which is transferred across the boundary; and it is to be regarded in the second place as introducing a distinction between the two methods of energy transfer - flow of heat and performance of work. Therefore, for Tolman, in view of the first aspect of the classical first law, it is consequent to consider the relativistic balance equations (2) for energy and momentum as its relativistic equivalent. But, to complete the analogy with the classical first law of thermodynamics, then one has still to introduce in relativistic thermodynamics a distinction between flow of heat and performance of work. This, however,

should be or, even, can only be made clear from a considerations of the relativistic second law of thermodynamics.

In concordance with these arguments, in relativistic TIP, energy of heat is not introduced as the fourth component of a vector, whose spacelike components are given by the heat flow. By setting, in analogy with non-relativistic TIP,

$$e = \rho c^2 + u \tag{30}$$

it is rather identified with the internal energy occurring in the energy-momentum tensor (20). The same is true for the 4-vector of the heat flow q^i: it appears as ingredient of the energy-momentum tensor, too. Finally, this provides equation (11). This is in accordance with the above-required considerations of the second law. - By the way, this path is also taken in non-relativistic TIP to reformulate classical thermodynamics locally to make it compatible with hydrodynamics.

For some authors, this is not completely convincing since, for them, the relativistic energy-momentum tensor and the corresponding conservation law concern only mechanical energy and not the energy of heat (see, e.g., [21]). This would mean that the relativistic energy-momentum tensor does not embrace the whole energy content of matter. However, then the theory of relativity should not be regarded as a universal theory to be unified with the other universally valid theory, thermodynamics, in an axiomatic manner. A similar objection says [11] that heat is a local form of energy, a fact which supposedly is not taken into consideration in standard relativistic TIP. Accordingly, it is proposed to split the energy-momentum tensor considered in relativistic TIP into mechanical and thermodynamical terms which should satisfy separated scalar conservation laws. Against this modification of relativistic TIP one must state: Relativity theory provides certain balance (or conservation) laws which we cannot arbitrarily change or supplement, without running into inconsistencies.

Thus, from the point of view of balance or conservation equations, one can only repeat the above said [20]: One has to take the relativistic first law as being merely a restatement of the principles of relativistic theory, while a clear understanding of the whole character of relativistic thermodynamics is only determined by the second law. According to these arguments, there seems to be no need for criticizing this part of relativistic TIP.

3.2. Structure of the entropy vector. As to the entropy 4-vector (21) and, thus, the formulation of the second law, one encounters some problems because there are ambiguities in the ansatz for the entropy vector. This follows from an identity derived in [22]. The identity reads

$$S^k = (s^k - \lambda q^k - \Psi n^k) + (\Psi N^k - \xi_l T^{lk}), \tag{31}$$

where

$$\lambda := \frac{1}{T}, \quad \Psi := \frac{1}{\rho}(s - e\lambda), \quad \xi_l := \lambda u_l. \tag{32}$$

(This identity is valid in any geometry and also for asymmetric energy-momentum tensors.) As a consequence of this expression [23], ansatzes for S^k found in literature can be shown to be only a special or even a wrong choice.

An example for a special choice is the ansatz (21): It results from (31) by assuming

$$\Psi = -\phi, \quad s^k = \frac{1}{T} q^k \quad (\text{or}, n^k = 0). \tag{33}$$

Another expression for the (equilibrium) entropy vector S^k is presupposed in [3, 4, 24]. It reads in [24]:

$$S^k = p\Theta^k - \phi N^k - \Theta_i T^{ik}. \tag{34}$$

However, this expression conflicts with the above identity, according to which the expression (21) can only supplemented by a spacelike, but not, as in (34), by a timelike vector.

Therefore, it should be useful to reconsider the results and, together with them, the resulting paradoxes with respect to possible generalizations or corrections of the respective ansatzes for the entropy vector.

3.3. Definition of the equilibrium. As above noted, in special-relativistic TIP one imposes a number of conditions on equilibrium states, where one of their implications is the Killing nature of the inverse-temperature vector Θ^k. In [25], conversely, first it is shown that the vanishing of shear σ^{ik} and expansion Θ can be deduced from the Killing equation:

$$\Theta_{i;k} + \Theta_{k;i} = 0. \tag{35}$$

Since the same cannot be done for the other equilibrium conditions the Killing equation appears only as a necessary, but not sufficient condition. However, the situation changes drastically if one assumes that the space-time is a Riemannian one which, via Einstein's gravitational equations, is curved by the matter fluid under consideration (see [25]). Then nearly all above-postulated equilibrium conditions can be derived.

In this case, the heat flow is given by the kinematic invariants (κ is Einstein's gravitational constant),

$$\kappa q_k = h_{kb}(\omega^{ba}{}_{;a} - \sigma^{ba}{}_{;a} + \frac{2}{3}\Theta^{;b}) + \frac{1}{c^2}(\omega_{ka} + \sigma_{ka})\dot{u}^a. \tag{36}$$

Assuming the validity of the Killing equation (34), for vanishing acceleration, $\dot{u}^k = 0$, one deduces all above required equilibrium conditions and, additionally, the restriction to states with $\rho + 3p = 0$. In the more general case, $\dot{u}^k \neq 0$, the Killing equation provides the following conditions:

$$\sigma_{ik} = 0, \quad \Theta = 0, \tag{37}$$

$$\dot{\rho} = 0, \quad \dot{p} = 0, \tag{38}$$

$$\Psi_l = T_{,m} h^m{}_l + \frac{T}{c^2}\dot{u}_l = 0 \Leftrightarrow \dot{u}^i = -c^2 \frac{T_{,l} h^{li}}{T}, \tag{39}$$

$$\kappa q^l = \omega^{la}{}_{;a} - \frac{2}{c^2} u^l \omega^2 + \frac{1}{c^2}\omega^{la}\dot{u}_a \quad (2\omega^2 = \omega_{ik}\omega^{ik}). \tag{40}$$

Eq. (39) shows that the acceleration of the fluid or in other words the non-geodesic movement is induced by the temperature gradient. Interestingly, here heat flow and anisotropic pressure do not vanish and eq. (39) has the same form as the relativistic generalization of

Fouriers law for vanishing heat-flow.. They are determined by the rotation ω_{ik} of the fluid and satisfy Cattaneo-like conditions,

$$h^m{}_a \dot{q}_m = -q^k \omega_{ka}, \tag{41}$$

$$h^m{}_a h^b{}_c \dot{\pi}_{bm} = 2\pi^k{}_{(a}\omega_{c)k}. \tag{42}$$

To solve the causality and instable-mode problems similar relations were proposed in [26] and open the field of different formulations of extended thermodynamics.

These remarks show that the usually required equilibrium conditions are too strong. They restrict the matter to non-rotating fluids. They are only justified for the linear Fourier ansatz providing with (41) vanishing q^l.

3.4. Balances with supply terms. Generally, the supply terms F^i and L^{ik} in eqs. (2) and (20) are not compatible with Einstein's gravitational equations,

$$R^{ik} - \frac{1}{2} R g^{ik} = -\kappa T^{ik} \tag{43}$$

if T^{ik} is put equal to ${}_m T^{ik}$ because these equations impose the following conditions on the energy-momentum tensor

$$T^{ik}{}_{;k} = 0, \quad T^{ik} = T^{ki} \tag{44}$$

Both conditions are implications of the structure of the left-hand side of the Einstein equations (43); it is symmetric and satisfies the (contracted) Bianchi identity. Thus, compatibility is only reached if F^i can be rewritten in the divergence of a tensor Θ^{ik} such that eqs. (44) are satisfied by the total tensor $T^{ik} = {}_m T^{ik} + \Theta^{ik}$.

An illuminating example is given in [2]. There, 4-force density F^i and couple L^{ik} are assumed to be of an electromagnetic nature what is indicated by the subscript "M". Exploiting results deduced from microscopical considerations it is shown that these terms can be written as

$${}_M F^i = -{}_M T^{ik}{}_{;k}, \quad {}_M L^{ik} = -{}_M T^{[ik]} \tag{45}$$

such that the total energy-momentum tensor

$$T^{ik} =_m T^{ik} +_M T^{ik} \tag{46}$$

satisfies the relations (44). This new energy-momentum tensor T^{ik} can be interpreted as the matter tensor of a magnetized and electrically polarized continuum [2]. It is compatible with the standard theory of general relativity. But it should be difficult to perform this procedure for all materials of interest in material theory.

Finally, it should be mentioned that one is not completely freed from this problem by going over to a so-called Einstein-Cartan theory of gravitation. In comparison to the theory of general relativity, this relaxes the problem only insofar as this theory does not require a symmetric energy-momentum tensor. However, in this theory one has two groups of field equations, where one of them contains the asymmetric energy-momentum tensor T^{ik} and the other one the spin tensor S^{ikl} as source terms. In virtue of these field equations and the Bianchi identities, T^{ik} and S^{ikl} satisfy certain balance equations. Again, only such supply terms are admissible which can be rewritten in a form that is compatible with these balance equations.

It should also be mentioned that relativistic thermodynamics generally does not sufficiently regard the constitutive equations and the related state spaces.

A discussion of the fundamentals of relativistic TIP is useful for different reasons, in particular, for the annoying paradox that the differential equations of the heat flow are of a parabolic character. Maybe, it can help to find the reason for and thus the way out of this dilemma. Another, but related ground is the hope that this could give some hint how to establish a relativistic thermodynamics far from equilibrium.

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