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A STRESS FIELD IN THE VORTEX LATTICE IN THE TYPE-II SUPERCONDUCTOR

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ABSTRACT. Magnetic flux can penetrate a type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes, or fluxons), each carrying a quantum of magnetic flux. These tiny vortices of supercurrent tend to arrange themselves in a triangular and/or quadratic flux-line lattice, which is more or less perturbed by material inhomogeneities that pin the flux lines. Pinning is caused by imperfections of the crystal lattice, such as dislocations, point defects, grain boundaries, etc. Hence, a honeycomb-like pattern of the vortex array presents some mechanical properties. If the Lorentz force of interactions between the vortices is much bigger than the pinning force, the vortex lattice behaves elastically. So we assume that the pinning force is negligible in the sequel and we deal with soft vortices. The vortex motion in the vortex lattice and/or creep of the vortices in the vortex fluid is accompanied by energy dissipation. Hence, except for the elastic properties, the vortex field is also of a viscous character. The main aim of the paper is a formulation of a thermoviscoelastic stress - strain constitutive law consisted of coexistence of the ordered and disordered states of the vortex field. Its form describes an auxetic-like thermomechanical (anomalous) property of the vortex field.

1. Introduction

The paper deals with a new phenomenological aspect of superconductivity. It develops the mechanics of a vortex lattice as a new state and geometry in a medium. The superconductors belong generally to two classes of such materials. A type-I superconductor expels magnetic flux from the material and hence is in the Meissner state. That is possible only at the applied magnetic field strength less than its determined critical value. In contrast, a type-II superconductor behaves in the other way. For applied field strength less than the lower critical field that superconductor will exhibit the usual Meissner effect. Applied fields greater than the upper critical field strength destroy the superconductivity altogether. In between the lower H_{c1} and upper H_{c2} magnetic field strengths the superconductor is in the mixed or vortex state. The second variable that determines the existence of that state is temperature $T < T_c$ (T_c denotes the critical phase transition temperature) [1-8].
Magnetic flux can penetrate a type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes or fluxons), each carrying a quantum of magnetic flux. These tiny vortices of supercurrent tend to arrange themselves in a triangular or quadratic flux-line lattice [9,10], which is more or less perturbed by material inhomogeneities that pin the flux lines. Pinning is caused by imperfections of the crystal lattice, such as dislocations, point defects grain boundaries, etc. Hence, a honeycomb-like pattern of the vortex

array presents some thermomechanical properties. In the natural state of any superconductor the termomechanical field comes from atom and/or molecular interactions both within crystalline (solid) and amorphous (fluid) states of the material in a presence of temperature changes. Such a situation transfers itself also to the vortex state. Since the vortices are formed by the applied magnetic field and around each of them the supercurrent flows, there exist also the Lorentz force interactions among them. Those interactions are an origin of an additional thermomechanical (stress) field occurring in the type-II superconductor. That field near the lower critical magnetic intensity limit H_{c1} is of the elastic character. However, if the density of the supercurrent is above its critical value and/or the temperature is sufficiently high, there occurs a flow of vortex lines in the superconducting body. Within such a situation vortices behave as a fluid rather than as an elastic lattice. The "fluidity" of the vortex array is also observed when the applied magnetic field tends to its upper critical limit H_{c2} [9,10]. In this way we meet a very interesting situation in a type-II superconductor. We can say, that two thermomechanical fields coexist in the medium. One of them is of a pure thermoelastic character coming from the mechanical properties of crystal lattice of the superconductor. The second one comes from the vortex array which, keeping its thermoelastic character near the lower magnetic field strength limit H_{c1}, transfers smoothly into a "fluid" near the upper magnetic field strength limit H_{c2}. The above phenomenon (transfer and coexistence) occurs in the $\{(H(T),T) : H_{c1} < H < H_{c2}, T < T_c\}$ space. However, just that second kind of field is of a peculiar (anomalous) character. Here it is, if the temperature T_0 within the entire normal state is equal to the "left" phase transition temperature (e.g. from superconducting to normal phase) the thermoelastic distortions in the stress are always negative, but if that temperature is equal to the "right" phase transition value (e.g. solid-fluid transition: melting point) those distortions are always positive within that phase. The thermoelastic distortions within the superconducting phase concerning only the vortex field behave differently. If the reference temperature T_0 - "left" is equal to zero $(0 < T_0 < T_c)$, they tend to $+\infty$. But if the reference temperature T_0 is equal to its "right" value T_c those distortions are always positive within the entire superconducting phase vanishing in T_c. The vortex field is also of the viscous character. The motion of vortices is damped by a force proportional to the vortex velocity. There are two reasons of that damping. The first one comes from simultaneous interactions among magnetic, mechanical and thermal fields. Then the second reason occurs because the resistivity in the area of vortex creep is the same as the resistivity of the current which would flow inside the vortex core. Hence the viscosity coefficient reads [4]

$$\eta = \frac{\Phi_0 \mu_0 H_{C2}}{\rho_n}, \tag{1}$$

where Φ_0 is the magnetic flux, μ_0 denotes the permeability of vacuum and ρ_n is the resistivity in the normal state. The paper aims at creation of a magnetothermomechanical model both for the "solid" and "fluid" states of the vortex field in such defined II-type superconductor. A specific definition of a stress tensor concerning the vortex field has been introduced within a constitutive theory based on proper representation of tensors, skew-symmetric tensors, vectors and scalars as functions of tensors, skew-symmetric tensors, vectors and scalars [11], theory being a part of a complete nonclassical thermodynamical model.

2. Thermodynamical foundations

Before we focus our attention on the main aim of the paper i.e. on a formulation of the thermoviscoelastic stress tensor for the vortex field (both for its lattice and fluid states), we recall a fundamental structure of the themodynamical model related to thermal and viscomechanical processes in the vortex array, the model as a background of our target considerations.

The structure of that model consists of [12]:

(1) vector of state - set of independent variables
(2) fundamental laws
    (a) balances
        - continuity equation
        - momentum balance
        - moment of momentum balance
        - internal energy balance
    (b) electromagnetic field equations
    (c) evolution equations of
        - fluxes
        - internal (hidden) variables (order parameters)
(3) entropy inequality
(4) vector of constitutive functions - set of dependent variables
(5) constitutive theory (a bridge between theory and reality) which determines a proper mapping between vector of constitutive functions and vector of state; it consists of laws of state, laws of processes and a residual inequality

One of the laws of state is just a mapping of the vortex stress tensor on the viscoelastic strain and temperature, the mapping which is the aim of the paper.

Following the properties and kind of phenomena listed in the previous section, the extended thermodynamical model for the viscoelastic field of vortices in the type-II superconductor is presented below. We have assumed that the mass density ρ of the vortex field concerns the density of the material in the normal state as the counterpart in the mixed type-II superconductor (i.e. the mass of the normal part of the body m^{normal} related to the total volume V of the material), and the energy dissipation occurs only because of the viscosity of the vortex field caused by the ohmic-like resistivity (normal-state resistivity) inside the vortex core [5]. Hence the general form of the state vector (the set of independent variables) reads (cf. [12,13])

$$C = \{\epsilon_{ij},\, \dot{\epsilon}_{ij},\, \varphi,\, A_i,\, T,\, T_{,i},\, \psi,\, \psi^*,\, \psi_{,i},\, \psi^*_{,i},\, q_i,\, j_i^S\} \tag{2}$$

where ϵ_{ij} denotes the strain tensor, its time derivative in (2) indicates the viscoelasticity of the vortex field, φ and A_i are the scalar and vector potentials, respectively, T is the absolute temperature, q_i is the heat flux, ψ is the order parameter (the wave function of a Cooper pair) and ψ^* is its complex conjugate, j_i^S is the supercurrent density. The fundamental laws, which govern set (2) are as follows

$$\dot{\rho} + \rho\nu_{k,k} = 0 \tag{3}$$

$$\rho\dot{\nu}_k - \sigma_{ik,i} - \varepsilon_{ijk}(j_i^N + j_i^S)B_j - f_k = 0 \tag{4}$$

$$\rho\dot{e} - \sigma_{ik}\nu_{k,i} - q_{k,k} - (j_i^N + j_i^S)\mathbb{E}_i - \rho r = 0 \tag{5}$$

$$\frac{1}{\mu_0}A_{i,kk} - j_i^N - j_i^S = 0 \tag{6}$$

$$\dot{q}_k - Q_k(C) = 0 \tag{7}$$

$$\dot{\psi} - \Psi(C) = 0 \tag{8}$$

$$\dot{\psi}^* - \Psi^*(C) = 0 \tag{9}$$

$$\dot{j}_k^S - J_k^S = 0 \tag{10}$$

ν_k denotes the velocity of the vortex field point, σ_{ik} is the viscoelastic stress tensor, j_k^N is the normal current, B_j is the magnetic induction, f_k is the body force, e is the internal energy density, $\mathbb{E}_i$ is the electromotive intensity, r is the heat source distribution. Set (3) to (10) consists of:

- the equation for vortex field whose form ensures the conservation of the vortex mass in the sense indicated above
- the momentum balance of the vortex field where elastic interactions are due to the Lorentz force
- the internal energy balance of the vortex field where the only dissipation occurs because of the Joule-like heat produced by the total current
- the electromagnetic vector potential equation
- the evolution equations for heat flux and supercurrent because of the extended thermodynamical model (2)
- the evolution equations for the Cooper pairs wave function as the order parameter (internal variable) evolution equations

The extended thermodynamical description has been chosen here since all the interactions run within low temperatures. Moreover, the electromagnetic field quantities satisfy the Maxwell equations, and the following relations hold

$$\mathbb{E}_i = E_i + \varepsilon_{ijk}\nu_j B_k \tag{11}$$

$$E_i = -\varphi_{,j} - \frac{\partial A_i}{\partial t} \tag{12}$$

$$B_i = \varepsilon_{ijk}A_{k,j} \tag{13}$$

$$B_i = \mu_0 H_i, \tag{14}$$

where H_k is the magnetic field strength. In the sequel we follow the assumption that φ vanishes by gauging [3].The use of the second law of thermodynamics in the form of the entropy inequality

$$\rho\dot{s} + \phi_{k,k} - \frac{\rho r}{T} \geq 0, \tag{15}$$

where s is the entropy density and ϕ_k denotes the entropy flux, gives us a possibility to determine all the constitutive functions which, in our case form the set

$$Z = \{\sigma_{ij},\ e,\ s,\ \phi_k,\ j_k^N,\ Q_k,\ \Psi,\ \Psi^*,\ J_i^S\} \tag{16}$$

As we have mentioned before, we are mostly interested in the mechanical properties of the vortex field vs. magnetic field $H_{C1} < H < H_{C2}$. Having several possibilities to create a proper constitutive theory we have chosen that based on representations of isotropic tensors, skew-symmetric tensors and vectors [11].

3. Stress in a thermo-magnetic vortex field

As we have mentioned before our aim was a constitutive model of thermo-magneto mechanical properties of the vortex field within entire phase region $\{H_{c1} < H < H_{c2}; T < T_c\}$ described by the stress-strain relation. If the general constitutive relation, basing on (2) and (16), is the following

$$Z = Z(C) \tag{17}$$

and we confine now to the situation

$$Z = \{\sigma_{jk}\} \tag{18}$$

and

$$C = \{\epsilon_{ij},\ \dot{\epsilon}_{ij},\ T,\ B_i\} \tag{19}$$

where $T < T_c$, we are able to find the looked for constitutive stress-strain relation in a thermomagnetic vortex array. The polynomial representation of a symmetric tensor as a function of symmetric tensors, axial vector and scalar reads [11,14]:

$$\begin{aligned} \sigma_{ij} = {} & \beta^1_\sigma \delta_{ij} + \beta^2_\sigma \epsilon_{ij} + \beta^3_\sigma \epsilon_{ik}\epsilon_{kj} + \beta^4_\sigma (B_i B_j - B_s B_s \delta_{ij}) + 2\beta^5_\sigma \varepsilon_{kjl}\epsilon_{ik} B_l + \\ & + (\beta^6_\sigma \varepsilon_{iks}\varepsilon_{ljp} B_s + 2\beta^7_\sigma \varepsilon_{ljp}\epsilon_{ik} + 2\beta^8_\sigma \varepsilon_{ikp}(B_j B_l - B_s B_s \delta_{lj}))\dot{\epsilon}_{kl} B_p + \\ & + \beta^9_\sigma \delta_{ij} + \beta^{10}_\sigma \dot{\epsilon}_{ij} + \beta^{11}_\sigma \dot{\epsilon}_{ik}\dot{\epsilon}_{kj} + 2\beta^{12}_\sigma \varepsilon_{kjl}\dot{\epsilon}_{ik} B_l + \\ & + (\beta^{13}_\sigma \varepsilon_{iks}\varepsilon_{ljp} B_s + 2\beta^{14}_\sigma \varepsilon_{ljp}\epsilon_{ik} + 2\beta^{15}_\sigma \varepsilon_{ikp}(B_j B_l - B_s B_s \delta_{lj}))\dot{\epsilon}_{kl} B_p + \\ & + 2\beta^{16}_\sigma \varepsilon_{ljp}\dot{\epsilon}_{ik}\dot{\epsilon}_{kl} B_p + \beta^{17}_\sigma (\epsilon_{ik}\dot{\epsilon}_{kj} + \dot{\epsilon}_{ik}\epsilon_{kj}) + \\ & + \beta^{18}_\sigma (\epsilon_{ik}\epsilon_{ks}\dot{\epsilon}_{sj} + \dot{\epsilon}_{ik}\epsilon_{ks}\epsilon_{sj}) + \beta^{19}_\sigma (\epsilon_{ik}\dot{\epsilon}_{ks}\dot{\epsilon}_{sj} + \dot{\epsilon}_{ik}\dot{\epsilon}_{ks}\epsilon_{sj}). \end{aligned} \tag{20}$$

Coefficients β^k_σ by the generators can be functions of the following set of invariants resulting from the vector of state (19):

$$\begin{aligned} & T,\ \epsilon_{kk},\ \epsilon_{ij}\epsilon_{ij},\ \epsilon_{ij}\epsilon_{jk}\epsilon_{kl},\ -2B_j B_j,\ \epsilon_{ij} B_i B_j,\ \epsilon_{kk} B_s B_s,\ \epsilon_{ij}\epsilon_{jk} B_i B_k, -\epsilon_{ij}\epsilon_{ij} B_s B_s, \\ & (\epsilon_{ij}\epsilon_{jk} B_k B_m - \epsilon_{ij}\epsilon_{jm} B_s B_s)\varepsilon_{nir}\epsilon_{mn} B_r,\ \dot{\epsilon}_{kk},\ \dot{\epsilon}_{ij}\dot{\epsilon}_{ij},\ \dot{\epsilon}_{ij}\dot{\epsilon}_{jk}\dot{\epsilon}_{kl},\ \dot{\epsilon}_{ij} B_i B_j,\ -\dot{\epsilon}_{kk} B_s B_s, \\ & \dot{\epsilon}_{ij}\dot{\epsilon}_{jk} B_i B_k,\ -\dot{\epsilon}_{ij}\dot{\epsilon}_{ij} B_s B_s,\ (\dot{\epsilon}_{ij}\dot{\epsilon}_{jk} B_k B_m - \dot{\epsilon}_{ij}\dot{\epsilon}_{jm} B_s B_s)\varepsilon_{nir}\epsilon_{mn} B_r, \\ & \epsilon_{ij}\dot{\epsilon}_{ij},\ \epsilon_{ij}\epsilon_{jk}\dot{\epsilon}_{kl},\ \epsilon_{ij}\dot{\epsilon}_{jk}\dot{\epsilon}_{kl},\ \epsilon_{ij}\epsilon_{jk}\dot{\epsilon}_{ks}\dot{\epsilon}_{si},\ \epsilon_{ij}\dot{\epsilon}_{jk}\varepsilon_{iks} B_s,\ \epsilon_{ij}\dot{\epsilon}_{jk}\varepsilon_{irs}\varepsilon_{krp} B_s B_p, \\ & \epsilon_{ij}\dot{\epsilon}_{ls}\varepsilon_{jkr}\varepsilon_{klp}\varepsilon_{sim} B_r B_p B_m,\ \epsilon_{ij}\dot{\epsilon}_{jk}\dot{\epsilon}_{ks}\varepsilon_{sir} B_r,\ \epsilon_{ij}\epsilon_{jk}\dot{\epsilon}_{ks}\varepsilon_{sir} B_r. \end{aligned} \tag{21}$$

Confining now the representation (20) to the linear form we have:

$$\sigma_{ij} = (\beta^1_\sigma(\epsilon_{kk}, T, B_k B_k) + \beta^9_\sigma(\dot{\epsilon}_{kk}, T, B_k B_k))\delta_{ij} + \beta^2_\sigma(B_k B_k)\epsilon_{ij} + \beta^{10}_\sigma(B_k B_k)\dot{\epsilon}_{ij} \tag{22}$$

To determine the coefficients β^k_σ we assume expression (22) to be of the similar form as for a thermo-visco-elastic isotropic body:

$$\sigma_{ij} = 2M\epsilon_{ij} + (L\epsilon_{kk} - \Lambda\Theta)\delta_{ij} + 2M\dot{\epsilon}_{ij} + L\dot{\epsilon}_{kk}\delta_{ij}. \tag{23}$$

The "material" coefficients M, L, M, L, Λ concern both the lattice and the fluid states of vortices. The linear approximation for σ_{ik} allows us to replace the temperature T to Θ, remembering that its values should not exceed the critical one T_c. Hence, we get the following definition of the relative temperature Θ:

$$\Theta = 0 \qquad if \qquad T = T_c$$

(24) $$\Theta = \frac{T - T_c}{T_c}, \quad 0 < T < T_c, \quad \longrightarrow \quad \Theta < 0$$

$$\Theta = -1 \qquad if \qquad T = 0.$$

This way the relative temperature within the superconducting phase is always negative. Remark that Θ cannot be defined by $T_c = T_0 = 0$. If so, the stress (23) should tend to infinity which contradicts physical sense.
To emphasise coexistence of the lattice- and fluid-like states we split σ_{ik} (23) into the trace and deviatoric parts as follows:

(25) $$\sigma_0 = \sigma_{kk},$$

(26) $$\tau_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_0\delta_{ij},$$

(27) $$\sigma_0 = (2M + 3L)\epsilon_{kk} - 3\Lambda\Theta + (2M + 3L)\dot{\epsilon}_{kk},$$

(28) $$\tau_{ij} = 2M\epsilon_{kk} + \frac{2}{3}M\epsilon_{kk}\delta_{ij} + 2M\dot{\epsilon}_{ij} + \frac{2}{3}M\dot{\epsilon}_{kk}\delta_{ij}$$

(29) $$\sigma_0 = \sigma_0^{lattice} + \sigma_0^{fluid}.$$

In order to determine the proper form of the looked for stress-strain relation we introduce now two parameters which determine actual state of vortices, i.e. if they are in the lattice, the fluid or the mixed state. Since just the magnetic field intensity decides on which state vortices the are, we propose those parameters as based on the first magnetic invariant in (21). In this way their dimensionless form is the following

(30) $$\alpha = \left(\frac{H_{c2} - H}{H_{c2} - H_{c1}}\right), \qquad \alpha = \begin{cases} 0 & if \quad H = H_{c2} \\ 1 & if \quad H = H_{c1}, \end{cases}$$

(31) $$\beta = \left(\frac{H - H_{c1}}{H_{c2} - H_{c1}}\right), \qquad \beta = \begin{cases} 0 & if \quad H = H_{c1} \\ 1 & if \quad H = H_{c2} \end{cases}$$

(32) $$\alpha + \beta = \begin{cases} 0 & if \quad H = H_{c1} \ or \ H = H_{c2} \\ f(H) & if \qquad H_{c1} < H < H_{c2}. \end{cases}$$

On defining now components (27) and (28) for the lattice and fluid states with the help of the parameters (30)-(32) we obtain:

(33) $$\sigma_0^{lattice} = \alpha(2\mu + 3\lambda)\epsilon_{kk} + \alpha(2\mu_L + 3\lambda_L)\dot{\epsilon}_{kk} - 3\lambda^T\Theta,$$

(34) $$\sigma_0^{fluid} = -3\beta\rho,$$

$$\tau_{ij}^{lattice} = 2\mu\alpha\epsilon_{ij} - \frac{2}{3}\mu\alpha\epsilon_{kk}\delta_{ij} + 2\mu_L\alpha\dot{\epsilon}_{ij} - \frac{2}{3}\mu_L\alpha\dot{\epsilon}_{kk}\delta_{ij}, \tag{35}$$

$$\tau_{ij}^{fluid} = \beta D\dot{\epsilon}_{ij}, \tag{36}$$

where the material-like coefficients responsible for the thermomechanical properties of the vortex field can be called as follows:

- λ, μ - Lame constants of the lattice,
- λ^T - thermoelastic constant of the lattice,
- λ_L, μ_L - viscoelastic constants of the lattice,
- p - pressure of the fluid,
- D - viscosity of the fluid.

As we have mentioned before we are mostly interested in mechanical properties of the vortex field vs. magnetic field $H_{c1} < H < H_{c2}$. Having several possibilities to create proper constitutive law for the stress tensor σ_{ij} (the main quantity of our interest), we choose it as based on isotropic polynomial representations of functions of tensor, vector and scalar variables [11]. Since our description is phenomenological and the vortex continuous field results from proper averaging with respect to the characteristic volume in macroscale, we assume that the considered lattice and fluid are isotropic. This is, of course, rough approximation (the vortex lattice is of the hexagonal symmetry) but sufficient to catch the transfer from the lattice to fluid up and to catch the viscoelastic and then viscous properties of the vortex field up. Following [15] the required form of the stress tensor σ_{ij} (p denotes the pressure of the vortex fluid) is:

$$\sigma_{ij} = \left[\left(\frac{1}{3}\alpha K - \frac{2}{3}\alpha G\right)\epsilon_{kk} - \frac{2}{3}\alpha\eta\dot{\epsilon}_{kk} - \lambda^T\Theta - \beta p\right]\delta_{ij}$$
$$+2\alpha G\epsilon_{ij} + 2(\alpha+\beta)\eta\dot{\epsilon}_{ij}, \tag{37}$$

where the elastic bulk (K) and shear (G) module are as follows:

$$3K = 2\mu + 3\lambda,\ G = \mu, \tag{38}$$

then viscous bulk (K_L) and shear (G_L) module are the following:

$$3K_L = 2\mu_L + 3\lambda_L,$$
$$G_L = \mu_L,$$
$$K_L = 3D = 3\eta,$$
$$G_L = D = \eta. \tag{39}$$

4. Numerical simulations for the vortex field stress in YBCO

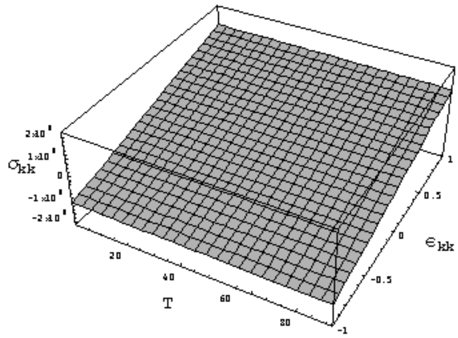

FIGURE 1. $\sigma_{kk}(T, \epsilon_{kk})$ for $B = 40T$

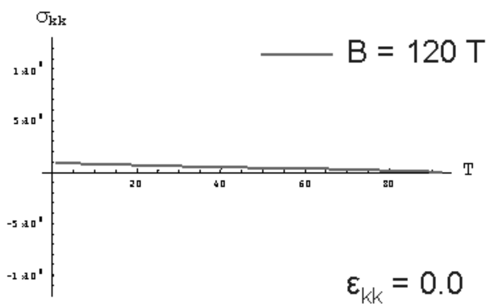

FIGURE 2. $\sigma_{kk}(T)$ for $\epsilon_{kk} = 0$ and $B = 120T$

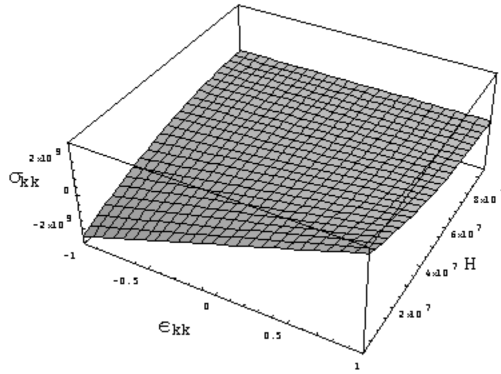

FIGURE 3. $\sigma_{kk}(H, \epsilon_{kk})$

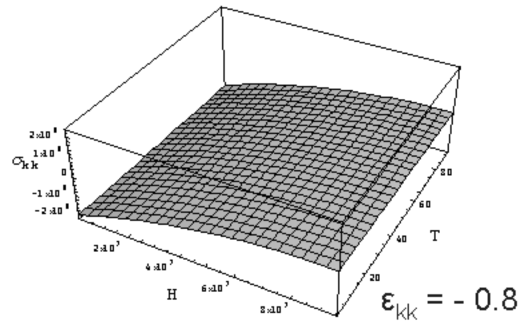

FIGURE 4. $\sigma_{kk}(H, T)$ for $\epsilon_{kk} = -0.8$

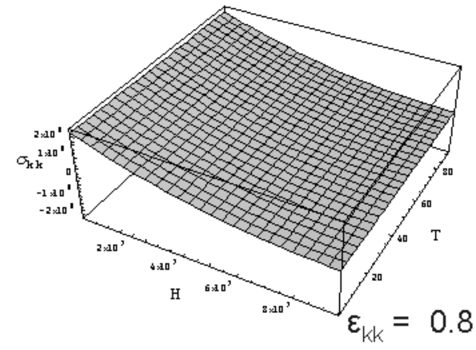

FIGURE 5. $\sigma_{kk}(H, T)$ for $\epsilon_{kk} = 0.8$

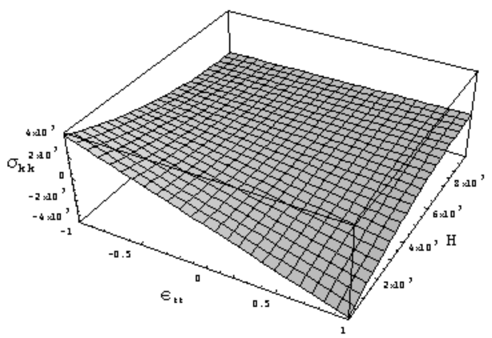

FIGURE 6. $\sigma_{12}(H, \epsilon_{12})$

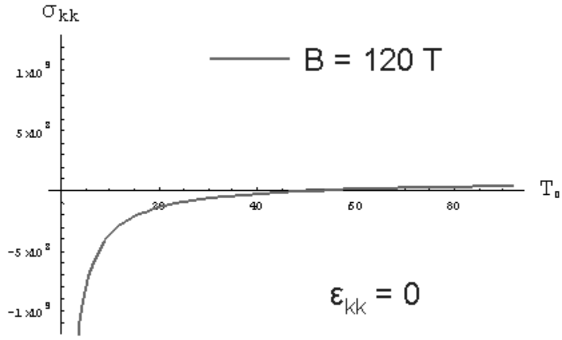

FIGURE 7. $\sigma_{kk}(T_0)$ for $0 < T_0 < T_c,\ T = 50K,\ \epsilon_{kk} = 0$

5. Conclusions

The vortex field from the thermomechanical point of view presents a very peculiar property. It seems to be of the auxetic-like one. The majority of materials being in the normal, non-superconducting state behave in such a way that if the temperature T_0 within the entire normal state is equal to the "left" phase transition temperature (e.g. from superconducting to normal phase) the thermoelastic distortions in the stress are always negative, but if that temperature is equal to the "right" phase transition value (e.g. solid-fluid transition: melting point) those distortions are always positive within that phase. That fact results from the experiment. However, from the thermodynamical model presented in the paper it results that within superconducting phase, where the vortex field exists, the thermoelastic distortions concerning only the vortex field behave differently from those occurring in materials being in the normal state. If the reference temperature T_0 - "left" is equal to zero $(0 < T_0 < T_c)$, they tend to $+\infty$ as absolute values. But if the reference temperature T_0 is equal to its "right" value T_c those distortions are always positive within the entire superconducting phase vanishing in T_c. The theoretical model in the form of the Clausius-Duhamel constitutive relation (the thermomechanical generalization of the Hooke law) confirms that fact.

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