DOI: 10.1478/C1V89S1P010

AAPP | Atti della Accademia Peloritana dei Pericolanti
Classe di Scienze Fisiche, Matematiche e Naturali
ISSN 1825-1242

Vol. 89, Suppl. No. 1, C1V89S1P010 (2011)

NOT JUST ENERGY, BUT MOMENTUM AND ANGULAR MOMENTUM TOO: MECHANICAL EFFECTS IN SCATTERING

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ABSTRACT. We review the transport and transfer of momentum and angular momentum by electromagnetic waves, and applications of the mechanical effects of scattering.

1. Introduction

The transport of energy by light is a widely-experienced fundamental aspect of electromagnetic waves, familiar to to all who use lasers, cook with microwaves, or perform the simple act of standing in sunlight.

What is less well-known is the transport of momentum and, especially, angular momentum. In concept, the transport of momentum and the consequent forces exerted by radition are old ideas, dating to at least Kepler's obervations of and speculations on comet tails [1]. With the development of improved experimental methods, attempts were made to measure such forces, for example by John Michell in the 1700s, a pioneer of the torsion balance, whose attempt failed to do other than demonstrate that concentrated sunlight can destroy one s experimental apparatus [2]. The forces involved are small, and success was not achieved until Lebedev [3] and Nichols and Hull [4, 5].

Astrophysical implications were realised, such as the Poynting Robertson effect, and the importance of radiation pressure in the support of stellar atmospheres against gravity. Terrestrial applications remained elusive, since the forces resulting from optical or electromagnetic radiation pressure are small compared to the power required, and are insufficient to overcome gravitational or frictional forces in most circumstances.

This changed in 1969 when Arthur Ashkin realized that, although the forces resulting from electromagnetic radiation were small, the forces required to move small particles were also small [6]. This prompted experiments demonstrating the acceleration and trapping of small particles by radiation pressure [7, 8].

The ability to non-invasively trap and manipulate microscopic particles made optical tweezers attractive to biologists, and the new possibility of measuring forces in microscopic biological systems was a major advance, enabled a wide variety of measurements in biophysics and other fields [9, 10, 11].

Meanwhile, rotational manipulation in optical tweezers was demonstrated by Sato [12], and led to a wide range of applications of rotation, including practical methods for microrheology [13, 14]. Some of these applications involved the design and production of microscopic objects specially designed to produce torque—optically-driven micromachines [15, 16].

2. Energy, momentum, and angular momentum

For a mathematical treatment of the transport of energy by electromagnetic fields, we can consider the work done on the currents within a volume by the electric field $\mathbf{E}$, and show energy flux density (ie, the power per unit area) is equal to the Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{1}$$

For a monochromatic wave, especially at optical frequencies, we are usually interested in the time-averaged Poynting vector in terms of the complex amplitudes rather than the real fields, which is equal to

$$\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^\star \tag{2}$$

where $\star$ denotes the complex conjugate. For a plane electromagnetic wave, the Poynting vector is always in the direction of propagation of the wave, and has a magnitude of

$$S = \frac{c\epsilon}{2n}|E|^2. \tag{3}$$

If the fields are known, this allows us to calculate the difference between the energy carried inwards by incoming radiation and the energy carried outwards by outgoing radiation, most simply done, for a steady-state case, in the far field. The difference between these two fluxes is the rate at which energy is transferred to the scatterer—usually the rate at which it is heated by the incident light. This is well-known, and widely taught.

The theoretical treatment of the transport of momentum was initiated by Maxwell s predictions of radiation pressure [17, 18], and followed by a thorough treatment a decade later by Poynting [19] and Heaviside. The general principles of transport of energy in media by waves were first formulated by Umov in 1874 (and one finds the Poynting vector called the Umov or Umov–Poynting vector in the Russian literature), who showed that the transport of momentum was a thermodynamically necessary consequence of the transport of energy, with the momentum flux density $\mathbf{p}$ determined by the energy flux density and the speed of transport of energy. The momentum transported per unit area in the direction of propagation of the wave is

$$\mathbf{p} = n\mathbf{S}/c. \tag{4}$$

The magnitude of the momentum flux is numerically equal to the energy density, since both are related to the energy flux by the same speed. From a modern perspective, this can be described in terms of the energy–momentum 4-vector, where the components corresponding the the momentum will be zero if the energy is stationary, and non-zero in a frame where the energy is moving.

Noting the dependence of the momentum flux on refractive index n in equation (4), we can recall the century-old Abraham–Minkowski controversy over the momentum of an electromagnetic wave in matter—whether the momentum flux density is $n\mathbf{S}/c$, $\mathbf{S}/(nc)$, or some other value. The relationships between energy density, flux, and transport speed resulting in (4) are valid for all waves (and indeed for other forms of transport of energy, such as thermal conduction etc.), and this momentum flux must be the total momentum flux associated with the electromagnetic wave [20].

Finally, any transport of momentum implies a transport of angular momentum as well. While it is tempting to simply write the angular momentum flux density $\mathbf{j}$ as the moment of the linear momentum flux density,

$$\mathbf{j} = \mathbf{r} \times \mathbf{p} = n\mathbf{r} \times \mathbf{S}/c, \tag{5}$$

this is not correct—electromagnetic waves can carry spin angular momentum, or *intrinsic angular momentum*—angular momentum that is independent of the choice of origin about which the moments are taken, and thus independent of $\mathbf{r}$. In general, the total angular momentum will be the sum of the spin $\mathbf{s}$ and orbital $\mathbf{l}$ angular momenta,

$$\mathbf{j} = \mathbf{l} + \mathbf{s}. \tag{6}$$

Curiously, however, while the moment of the linear momentum density cannot be the angular momentum density, it is known that for fields finite in extent, the total angular momentum is given by the integral over the entire field of the moment of the linear momentum density. This integral can also be written in terms of spin and orbital angular momentum densities [21]. Unsurprisingly, this has led to yet another century-old controversy, with the fires kept well-stoked by the fuel of optical rotation in laser trapping [22, 23]. For practical purposes, it should suffice to note that there is a clear physical distinction between spin and orbital angular momenta for time-harmonic waves the spin can be determined from the Stokes parameters, and the orbital angular momentum cannot [24].

3. Conclusion

The transport of energy, momentum, and angular momentum by electromagnetic fields can be directly calculated from the fields, if the fields are known. Therefore, the absorbed power, the force, and the torque resulting from scattering can be calculated if the fields can be calculated—that is, if the problem at hand is amenable to computational modelling. This can enable a new level of theoretical understanding, and allows the quantitative design and engineering of optically-driven micromachines. In turn, this has driven work on the modelling of a range of complex structures [25]. In addition, a range of new technologies, especially involving rotation, are reaching the stage of practical use [11]. The story, of course, does not end here, as this is an active and developing field, although our review of this field must.

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Paper presented at the ELS XIII Conference (Taormina, Italy, 2011), held under the APP patronage; published online 15 September 2011.