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ANGULARLY RESOLVED REFLECTANCE FROM RANDOM AND ALIGNED SEMI-INFINITE MEDIA

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ABSTRACT. The angularly resolved reflectance from semi-infinite turbid media is investigated using the Monte Carlo method. Turbid media with random and aligned microstructures are considered. It is shown that in the case of random media the Lambert law is valid for one special configuration of the optical and geometrical properties. For random media having an aligned microstructure the angularly resolved total reflectance is completely different from a Lambertian characteristic.

1. Introduction

The Lambert law is often used for characterizing the angularly resolved reflectance from an illuminated turbid medium. It states that the reflectance is proportional to $\cos(\theta)$, where θ is the polar angle relative to the normal of the medium's boundary [1, 2, 3]. We very recently showed that for a random turbid medium the angularly resolved total reflectance is only exactly Lambertian for a special set of the optical properties of the random medium and for a special illumination geometry [4]. Here, total reflectance means that the reflectance integrated over all locations is considered. In this study we explore the reason for this in more detail, especially the dependence of the total reflectance on the depth of an isotropic source is investigated. Further, it is shown that the Lambert law is not fulfilled for any deviation from this special parameter set. In addition, a large difference to the Lambert law is found when not only the polar dependence but also the azimuthal dependence of the reflectance is considered. Finally, it is reported that the total reflectance from an aligned turbid medium is very different from the Lambert law.

2. Methods

Within the framework of classical physics light propagation in scattering media is exactly described by the Maxwell's equations. Due to the high demand on the CPU and the memory of the computer it is only possible to calculate relative small volumina ($<<$ 1mm^3) for complex media, e.g. biological tissue, using this approach. Thus, the radiative transfer equation or its approximations are often applied to describe the light propagation in larger volumes of scattering media. Here, we use the Monte Carlo method to numerically solve the radiative transfer equation. It was recently shown that the Monte Carlo method equals, in the limit of an infinitely large number of used photons, exactly analytical solutions of the transport equation, which we derived for the fluence and the radiance of an infinitely extended turbid medium that is illuminated by e.g. an isotropic point source [5].

A Monte Carlo method was implemented which is not only capable to describe the light propagation of random media, e.g. consisting of spheres, but also of aligned random media, e.g. consisting of cylinders which are more or less aligned in a certain direction. To this end the scattering functions of the single scatterers (e.g. cylinders) for all incident angles were calculated by solving the Maxwell's equations and by using these functions in the Monte Carlo simulations. It was assumed that the semi-infinite turbid medium is separated from the non-scattering medium by a plane boundary. When a photon hits the boundary the Fresnel equations were used to calculate the probability for reflectance and transmittance. For describing the scattering functions of the turbid media which exhibit an isotropic light propagation the Henyey-Greenstein function or the Mie theory were applied.

3. Results

3.1. Total reflectance versus polar angle. The total reflectance from a semi-infinite medium versus the polar angle $R(\theta)$ was calculated using the Monte Carlo method. The optical properties (the absorption coefficient μ_a, the scattering coefficient μ_s, the scattering function, and refractive index n) and the angular distribution of the illuminating light were systematically altered. It was found that an exact Lambert law is obtained when following three conditions are fulfilled:

1) The scattering is infinitely larger than the absorption ($\mu_s \neq 0, \mu_a = 0$)

2) The refractive indices of the scattering and the non-scattering media are matched

3) The angular distribution of the incident light is Lambertian

Figure 1 shows the total reflectance divided by $\cos(\theta)$ using the above three conditions and an isotropic scattering functions (black curve) or an Henyey-Greenstein scattering function with an anisotropy factor $g = < \cos(\theta) > = 0.95$ (red curve). It can be seen that within the noise of the Monte Carlo simulations the Lambert law is fulfilled. In additon, figure 1 shows $R(\theta)/\cos(\theta)$ for these conditions using an isotropic scattering function for a non-zero absorption (green curve), for a perpendicular incident light (brown curve), and for a mismatch of the refractive index (blue curve) demonstrating that, if one of the above mentioned three conditions is changed, deviations from the Lambert law are obtained.

The green curve shows a decrease of the reflectance at all polar angles compared to the Lambert law due to the non-zero absorption. The decrease is larger at small polar angles compared to large polar angles. This means that the photons remitted at large polar angles propagated a smaller path through the scattering media, because thereby the probability of absorption is smaller. This was further investigated in figure 2, where $R(\theta)/\cos(\theta)$ is shown for isotropic sources which are located at different depths below the boundary of the

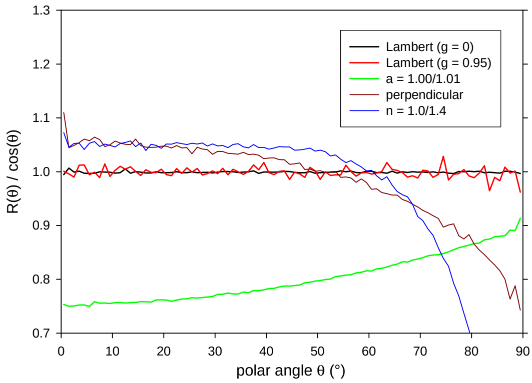

Figure 1. $R(\theta)/\cos(\theta)$ versus $\cos(\theta)$ for different optical and geometrical properties as indicated in the legend.

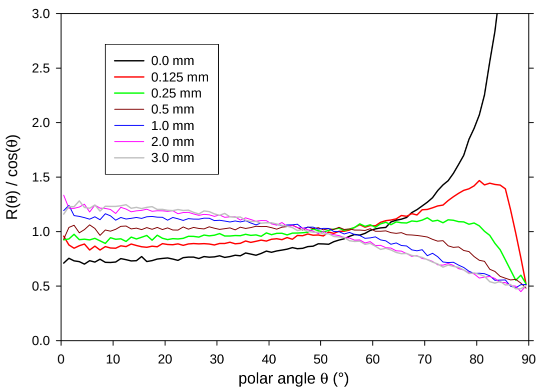

Figure 2. $R(\theta)/\cos(\theta)$ versus $\cos(\theta)$ for isotropic point sources which are located at different depths below the boundary between the scattering and the non-scattering medium as indicated in the legend. The optical parameters are: $\mu_s/\mu_a = \infty$, isotropic scattering, and matched refractive indices.

scattering medium. When the isotropic source is located directly at the boundary the probability of reflectance at large polar angles is larger than for an exact Lambert surface. At deeper depths of the isotropic sources, however, the probability for reflectance at smaller polar angles is larger. This is caused by the different probability that a photon generated at the source leaves the turbid medium without further scattering. For source located sufficiently deep under the surface the probability of a photon being reflected without scattering interaction approximates zero. This is the reason why $R(\theta)/\cos(\theta)$ converges to the same values for larger source depths, see Fig. 2.

3.2. Reflectance versus polar and azimuthal angles. The reflectance from turbid media versus polar and azimuthal angles were investigated for a point source which is incident onto random media. Huge deviations from the Lambert characteristic were found especially at small distances from the incident source. In addition, significant deviations even occur at large distances ($>> (\mu_s(1-g))^{-1}$) from the incident source. For a lot of applications in the field of biomedical optics the azimuthal dependence has been neglected.

3.3. Reflectance from media having an anisotropic light propagation. For aligned turbid media which exhibit an anisotropic light propagation even the total reflectance shows large deviations from the Lambert law. We performed angular resolved measurements on wood and found that almost all reflected light is concentrated in a plane perpendicular to the growth direction of the wood. A Monte Carlo method considering the scattering of the microstructure of the wood was developed and showed excellent agreement with the measurements [4]. In these calculations the microstructure of wood was approximated by aligned cylinders and, in addition, randomly aligned scatterers.

4. Conclusions

We showed using a numerical solution of the radiative transfer theory that only for a special set of optical and geometrical parameters the reflectance of an illuminated turbid media is exactly Lambertian. For many relevant applications the reflectance has a large deviation from the simple Lambert law.

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