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DIFFRACTION OF A PLANE WAVE ON A MULTILAYERED GRATING

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ABSTRACT. The two-dimensional problem of wave scattering on a multilayered grating consisting of dielectric infinite cylindrical bodies with arbitrary cross-section is considered. The system of integral equations to which the initial problem is reduced is derived. The efficient algorithm for calculation of periodic Green's function is offered. The dependencies for reflected and transmitted field are obtained.

1. Introduction

The paper considers two-dimensional problem of diffraction of a plane wave on a multilayered periodical grating. This problem is interesting in a wide class of areas, for example the structure simulates photonic crystals. To solve the problem we use a modified null field method (MNFM), which has previously been successfully approved in [1, 2]. The null field method (NFM) has been offered for the first time by Waterman [3]. The modification of NFM named in the literature also as a method of T-matrix is successfully used to solve a wide class of diffraction problems. The basis for the NFM is some relation which is obeyed everywhere inside the scatterer [1, 2]. If we require that this relation is fulfilled on some closed surface inside the scatterer the initial boundary problem is reduced to the integral equation of the first kind relative to unknown current distributed on the surface of the body. In papers [1, 2] it has been shown that to develop the most high-speed and stable algorithms the auxiliary surface should be constructed by means of analytical deformation of the surface of the scatterer. In [1, 2] this version of NFM is called the modified null field method.

Note also that, in the problem of wave scattering by a periodic grating considered below, the periodic Green's function is used. Computation of this function is hampered by certain factors. Here, the periodic Green's function is calculated by two techniques [4]. In the case of large distance (along coordinate which is perpendicular axis of the grating) between point of source and point of observation, one can use a series obtained by the Poison formula. In the case when the distance is small it is possible to expand the Green's function into a series of cylindrical harmonics.

2. Derivation of the main relations

Consider the periodical grating consisting of several rows of infinite cylindrical bodies, whose generatrices are parallel to the axis z. We denote the number of rows by M. Suppose that the period of each layer is equal to d_j where $j = 1, 2, \dots M$. Note that the bodies of different layers can differ from one another. Let S_{0j} is the contour of the central element of *j*-th layer. We introduce the local coordinate system (x_j, y_j) connected with the central element of *j*-th row of the grating (the coordinates of general system are $x = x_1, y = y_1$). Denoted by $\vec{r}_{lj} = (p_{lj}, q_{lj})$ is the position vector directed from the origin of *j*-th row to the origin of *l*-th one. Allow that the structure is irradiated by the plane wave

$$u^0 = \exp(-ikr\cos(\varphi - \varphi_0)) \tag{1}$$

where (r, φ) are the polar coordinates, k is the wave number, φ_0 is the incidence angle of the plane wave. The diffraction field satisfies the radiation condition at infinity

$$u^1(x, y) = \sum_{l=1}^{M} \sum_{s=-\infty}^{\infty} A_{ls}^{\pm} \exp(-iw_{ls}x_l \mp i\nu_{ls}y_l) \tag{2}$$

where $w_{ls} = \frac{\chi_l + 2\pi s}{d_l}$, $\nu_{ls} = \sqrt{k^2 - w_{ls}^2}$, $\chi_l = kd_l \cos\varphi_0$. Here the sign of square root is chosen so that its imaginary part is not positive. We consider the following continuity conditions is to be fulfilled at the contours S_{0j}

$$\begin{aligned} &u = u^i \\ &\frac{\partial u}{\partial n} = \frac{1}{\eta_j}\frac{\partial u^i}{\partial n} \end{aligned} \qquad \eta_j = \begin{cases} \mu_j^i/\mu, & \text{E-polarization} \\ \varepsilon_j^i/\varepsilon, & \text{H-polarization} \end{cases} \tag{3}$$

where u^i is the field inside the central element of j-th layer, $\frac{\partial}{\partial n}$ is the derivative along the outward normal to S_{0j} and $\varepsilon, \mu, \varepsilon_j^i, \mu_j^i$ are the characteristics of the media outside and inside the elements of the grating respectively.

Let's apply MNFM. Specify the contours S_{0j} in the local polar coordinate system:

$$x_j = \rho_j(\varphi_j)\cos\varphi_j, \quad y_j = \rho_j(\varphi_j)\sin\varphi_j \tag{4}$$

Then we introduce the auxiliary contours $\Sigma_{0j}^{\pm}$ (inside and outside S_{0j}) as follows

$$x_j^{\pm} = \rho_j^{\pm}\cos\alpha_j^{\pm}, \quad y_j^{\pm} = \rho_j^{\pm}\sin\alpha_j^{\pm} \tag{5}$$

where

$$\alpha_j^{\pm} = \arg\xi_j^{\pm}(\tau), \quad \rho_j^{\pm} = |\xi_j^{\pm}(\tau)|, \quad \xi_j^{\pm}(\tau) = \rho_j(\tau \pm i\delta_j^{\pm})\exp(i\tau \mp \delta_j^{\pm}) \tag{6}$$

The upper sign in the formulas (6) corresponds to the contour Σ_{0j}^{+}. The values δ_j^{+} and δ_j^{-} are the positive parameters responsible for the degree of deformation of the contour S_{0j} and $\tau \in [0, 2\pi]$. The choice of δ_j^{+} and δ_j^{-} is detailed in [1, 2]. In accordance with MNFM we state the following conditions at the auxiliary contours $\Sigma_{0j}^{\pm}$:

$$\sum_{l=1}^{M} \int_{S_{0l}} \left[u_l(\vec{r}'_l)\frac{\partial G_l(\vec{r}_j, \vec{r}'_l)}{\partial n'_l} - v_l(\vec{r}'_l)G_l(\vec{r}_j, \vec{r}'_l) \right] ds'_l = -u^0(\vec{r}_j) \tag{7}$$

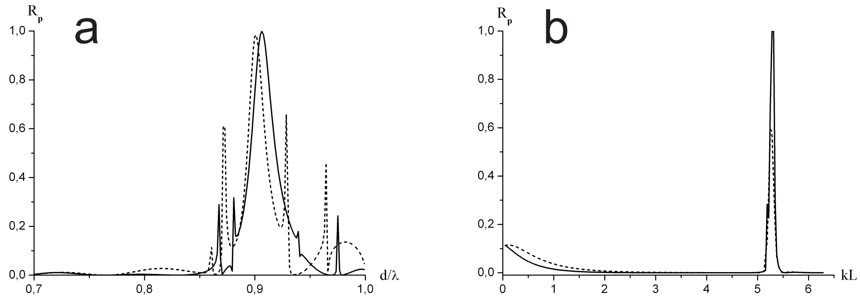

Figure 1. (a) The dependence of power reflection coefficient for the grating from identical layers. (b) The dependence of power reflection coefficient for hexagonal grating.

$$\int_{S_{0j}} \left[u_j(\vec{r}\,'_j) \frac{\partial G^i_j(\vec{r}_j, \vec{r}\,'_j)}{\partial n'_j} - \eta_j v_j(\vec{r}\,'_j) G^i_j(\vec{r}_j, \vec{r}\,'_j) \right] ds'_j = 0 \tag{8}$$

where $\vec{r}_j \in \Sigma^+_{0j}$ in Eq. (7) and $\vec{r}_j \in \Sigma^-_{0j}$ in Eq. (8) $(j = 1, 2, \ldots M)$. We denote $v_l = \frac{\partial u_l}{\partial n'_l}$ and

$$G_l(\vec{r}_j, \vec{r}\,'_l) = \frac{i}{4} \sum_{l=-\infty}^{\infty} H^{(2)}_0 \left(k\sqrt{(x_j - x'_l + p_{lj} - sd_l)^2 + (y_j - y'_l + q_{lj})^2} \right) \times \\ \times \exp(-is\chi_l) \tag{9}$$

$$G^i_j(\vec{r}_j, \vec{r}\,'_j) = \frac{i}{4} H^{(2)}_0 \left(k^i_j \sqrt{(x_j - x'_j)^2 + (y_j - y'_j)^2} \right) \tag{10}$$

Thus the problem is reduced to solving the system of integral equations of the first kind relative unknown currents on the contours S_{0j}. The system (7), (8) is solved numerically using collocation technique. The way of calculation of the values $G_l(\vec{r}_j, \vec{r}\,'_l)$ is described in [4].

3. Numerical results

Figure 1a illustrates the frequency dependence of power reflection coefficient (i.e. the value $R_p = k \sin\varphi_0 |R_0|^2$, where R_0 is the standard reflection coefficient of zero mode) for the six-layer grating consisting from the identical cylinders with circular cross-section or cylinders with superelliptic cross-section. The superelliptic contour is described by the equation

$$\left(\frac{x}{a}\right)^{2q} + \left(\frac{y}{b}\right)^{2q} = 1 \tag{11}$$

At high magnitude of the parameter *q* the grating of such geometry is little different from the grating consisting from rectangular elements. The solid curve in fig. 1a corresponds to the scattering on the grating formed by circular cylinders and the dashed curve corresponds to diffraction on the grating formed by the superelliptic cylinders.

The relative radius of the circular elements is $a/d = 0.3$ and the values $a/d = 0.3$, $b = a, q = 10$. The distance between the layers is $h/d = q_{12}/d = 0.7$. The other parameters are $\mu = \mu_i = 1, \varepsilon = 1, \varepsilon_i = 2$ and $\varphi_0 = 90°$. We compared our results for the grating of circular cylinders with those presented in paper [5]. The results agree with good accuracy.

Figure 1b illustrates the dependence of power reflection coefficient for diffraction of the plane wave on the hexagonal six-row grating. On an axis of abscises the value kL i.e. the distance between the elements of the grating is plotted. The lattice geometry is that the horizontal coordinate of odd layers is equal to a half of the period of the grating. We have considered the grating consisting of circular elements (solid curves in the figure) and the elements with superelliptic cross-section (dashed curves). The sizes of the elements of the grating and the distance between the rows are $2a = 0.15\lambda$, $h = 3\sqrt{3}\lambda/8$ ($a = b$ and $q = 10$ for superelliptic elements). The period of the grating $d = 0.75\lambda$ and $\mu = \mu_i = 1$, $\varepsilon = 1$, $\varepsilon_i = 2.25$, $\varphi_0 = 90°$.

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