

## ON RETRIEVING SHAPE INFORMATION FROM SCATTERING PHASE MATRICES USING A DISTRIBUTION OF SPHEROIDS

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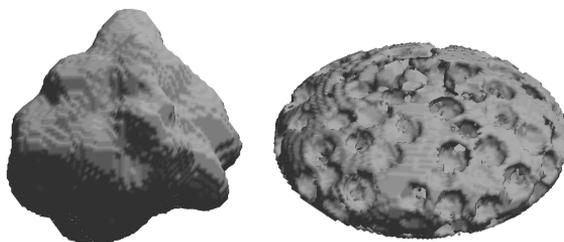
**ABSTRACT.** Phase matrices of spheroids are fitted to phase matrices of reference particles with identical sizes and refractive indices to test how well spheroids can be used to retrieve shape information from less symmetric target particles. It is found that shapes of best-fit spheroids do not correlate well with shapes of the reference particles even when the phase matrix is reproduced well by the spheroids.

### 1. Introduction

Particle morphology is one of the key factors affecting how electromagnetic radiation is scattered by single particles. Due to its simplicity and high efficiency, the homogeneous sphere model has been very popular, even though it is well known to produce errors when applied to nonspherical targets. Lately, one of the most popular replacement for spheres has been the homogeneous spheroid model, for which light scattering can be solved exactly and efficiently using  $T$ -matrix methods [1]. Indeed, a shape-size distribution of spheroids have proven to mimic scattering by nonspherical dust particles better than some approaches based on more realistic particle shapes (e.g., [2, 3]). One aspect that has received little attention is why the spheroids work so well.

There are two plausible explanations for this. The most obvious explanation would be that spheroids scatter light very similarly to particles with irregular shapes and similar aspect ratios. The second explanation would be that different spheroids scatter light so differently from each other that, as a distribution, they can be fitted well to many types of scattering patterns. In the latter case, the physical properties of the best-fit spheroids would not necessarily correlate in any clear way with the target particles.

Considering that spheroids are quickly becoming a standard replacement for spheres in many remote sensing applications, it seems that this issue should be addressed. To this end, we compute phase matrices for a number of reference shapes, fit the phase matrices with a distribution of spheroids, and compare the obtained shape distribution with the shape of the target particle.



**Figure 1.** The reference shapes considered in this abstract. On the left, a Gaussian random sphere, and on the right, a porous oblate spheroid with  $\xi = 1.0$ .

## 2. Modelling approach

DDSCAT, a discrete-dipole-approximation model by [4] is used for computing the phase matrices for the reference particles. Phase matrices for model spheroids are computed using the  $T$ -matrix method by [1]. Model spheroids are fitted to the reference phase matrices using the Levenberg-Marquardt method [5]. We assume a refractive index of  $m = 1.5 + i0.002$  for both the reference particles and model spheroids, and compute scattering for size parameters from  $x = 0.5$  to 10 with increments of 1.0 for  $x$  above unity. A lognormal size distribution with  $r_g = 0.35 \mu\text{m}$  and  $\sigma = 1.8$  (see [1]) is used for size averaging.

The model spheroids cover a shape parameter range from  $\xi = -2.0$  to 2.0 with increments of 0.25. The shape parameter is defined as

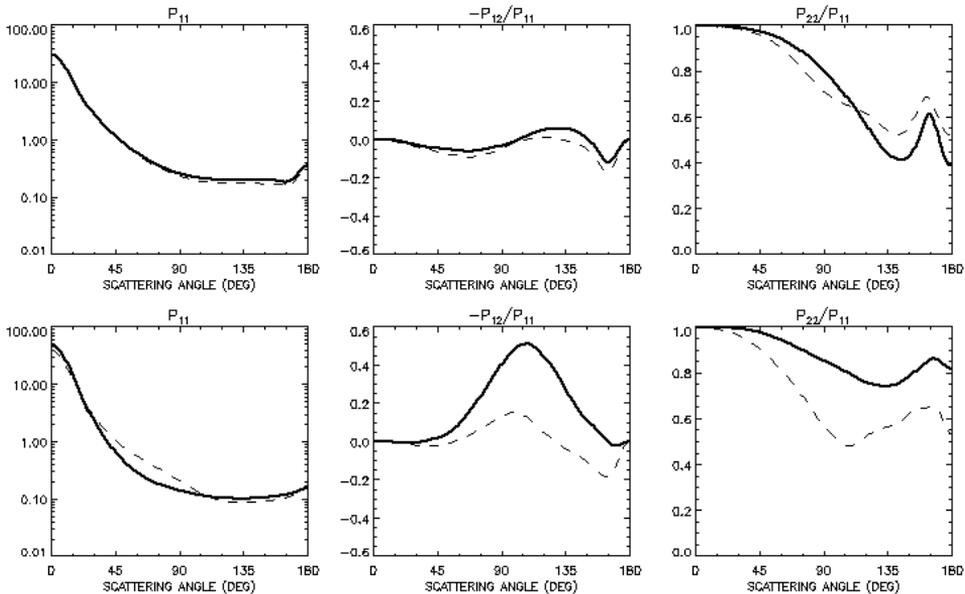
$$\xi = \begin{cases} b/a - 1 & a \leq b \text{ (oblate)}, \\ 1 - a/b & a > b \text{ (prolate)}, \end{cases} \quad (1)$$

where  $a$  is the diameter of the spheroid along its main symmetry axis, and  $b$  the maximum diameter in the orthogonal direction.

Several reference targets have been used, but here we only consider two cases in detail: a single Gaussian random sphere and an oblate spheroid filled with spherical cavities (Fig. 1). These synthetic targets are not considered to be better models for scattering by real non-spherical particles, but merely example shapes to test the shape inversion.

## 3. Results

The phase matrix of the Gaussian random sphere could be well reproduced with the model spheroids, while for the porous oblate spheroid the fit was clearly worse (Fig. 2). For a cube, another compact shape tested, the fits were somewhat worse than for the Gaussian random sphere (not shown), but still clearly better than for the porous targets. Fits for a porous prolate spheroid with  $\xi = -0.5$  (not shown) were similar to those for the porous oblate spheroid. Phase matrices of compact particles thus appear to be easier to mimic than those of porous targets with a distribution of spheroids. Interestingly, for porous targets the fits were better when outer-dimension-equivalent sizes were used instead of the volume equivalence. A slight improvement in the fits was obtained also when the Maxwell-Garnett



**Figure 2.** Comparison of the phase matrix elements  $P_{11}$ ,  $-P_{12}/P_{11}$ , and  $P_{22}/P_{11}$  for the reference particle (solid line) and that from the best-fit spheroids (dashed line). The upper row is for the Gaussian random sphere and the lower row for the porous oblate spheroid.

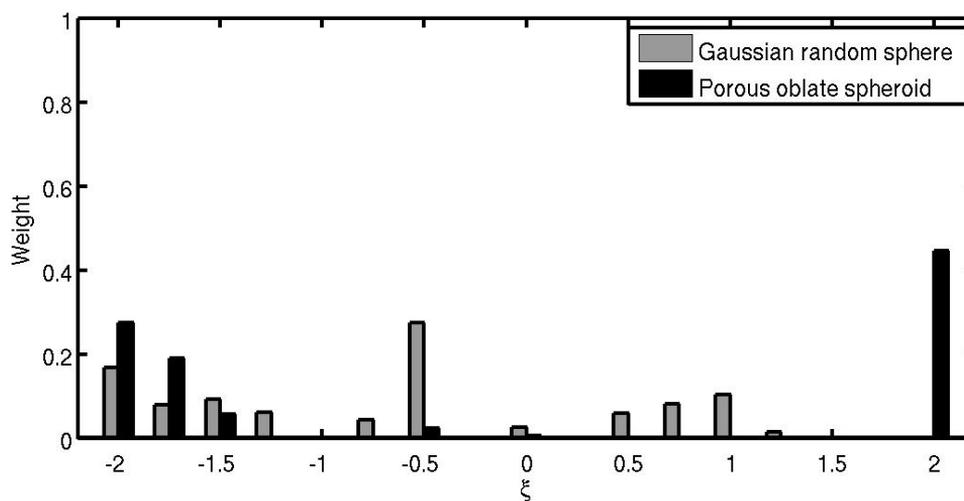
effective medium approximation was used to account for the porosity. Nevertheless, the performance for porous targets was clearly inferior compared to the compact targets.

The shape distributions corresponding to best fits for the Gaussian random sphere and the porous oblate spheroid are shown in Fig. 3. For the Gaussian random sphere, the shape distribution peaks at  $\xi = -0.5$  which is a reasonable value. However, more than half of the model spheroids are widely distributed over other aspect ratios. For the cube (not shown), the best-fit shape distribution is strongly dominated by two aspect ratios,  $\xi = -2.0$  and  $0.5$ , neither corresponding to a cube. For the porous oblate spheroid, the shape distribution retrieved is dominated by extreme-aspect ratio oblate and prolate spheroids. For the porous prolate spheroid with  $\xi = -0.5$  (not shown), the retrieved shape distribution consists almost completely of extreme-aspect ratio prolate spheroids.

To conclude, the best-fit shape distributions do not match the shape of the target particle even when the phase matrix of the reference particle is reproduced well, and the shape distributions retrieved show little correlation with the actual target shapes.

## Acknowledgments

The research has been funded by the Academy of Finland (Contract 125180) and the Swedish Research Council (Contract 80438701).



**Figure 3.** The best-fit shape distributions for the Gaussian random sphere and the porous oblate spheroid.

## References

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Paper presented at the ELS XIII Conference (Taormina, Italy, 2011), held under the APP patronage; published online 15 September 2011.

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