

DECOMPOSITION OF MUELLER MATRICES OF SCATTERING MEDIA: THEORY AND EXPERIMENT

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ABSTRACT. Algebraic decomposition of Mueller matrices is a particularly promising approach to the retrieval of the optical properties of the medium investigated in a polarized light scattering experiment. Various decompositions of generally depolarizing Mueller matrices are revisited and discussed. Both classic as well as recently proposed approaches are reviewed. Physical and mathematical aspects such as depolarization and limits of applicability are comparatively addressed. Experimental matrices of scattering media are decomposed by different methodologies and physically interpreted.

Mueller matrices represent the total information one is able to obtain from the interaction of polarized radiation with a linear-response medium. In particular, their sixteen real elements are the complete phenomenological descriptors of a polarized light scattering experiment.

With the ready availability of Mueller matrix polarimeters and the continuously increasing complexity of the scattering media under investigation, the interpretation of an experimentally obtained Mueller matrix in terms of physical properties of the medium is of ever growing importance today. Indeed, in numerous practical cases it is not possible to directly relate the polarimetric response of the medium to its elementary properties (dichroism, birefringence, optical activity...) through rigorous electromagnetic (EM) theory modelling. An additional (and serious) complication to be handled within the EM-modelling approach is the possible presence of depolarization, i.e. the partial loss of coherence (or decorrelation) of the probing light as a result of its interaction with the medium. An alternative approach to this problem is algebraically decomposing the experimental Mueller matrix into simpler components without any explicit reference to an EM-scattering model.

More specifically, the product algebraic decompositions [1, 2] represent a generally depolarizing Mueller matrix as a product of the Mueller matrices of "elementary" optical components such as diattenuators, retarders and depolarizers. The potential benefit of applying the algebraic approach to experimentally obtained Mueller matrices e.g., from biological samples, is twofold. First, the algebraic methodology is universal, in contrast to modelling the optical response of the sample. That is, algebraic decompositions are applicable to any experimental Mueller matrix, whether an EM model describing the medium under investigation exists or not. This allows the experimentalist to obtain immediate physical information on the sample - through the representation of the latter as a chain of elementary optical components - even in the absence of any optical model or (most often)

when the latter is either too complex or not accurate enough. As an example, the so called symmetric decomposition separates the polarizing from the depolarizing properties of the original Mueller matrix and furthermore, allows one to determine where the depolarization has taken place [3]. The second advantage of the algebraic approach stems from the "standard" representation of every Mueller matrix in a canonical form [4] playing the role of an "optical equivalent circuit" having the same polarimetric response as the original matrix. The equivalent circuit approach thus makes it possible to perform a formal comparison - in terms of polarimetric properties - of Mueller matrices of various physical origins. For instance, the application of algebraic methods on Mueller matrices of biological samples - well known for their rich and often too-complex-to-be-modelled polarimetric response - opens up the way of novel analysis possibilities potentially allowing for a better physical comprehension [5, 6].

Eventually, another algebraic approach, proposed recently as a complementary alternative to the product decomposition one, is the differential matrix decomposition [7]. A natural generalization of the classic differential matrix formalism [8] to the depolarizing case, this approach, based on the physical picture of a continuously distributed depolarization, parallels and complements the product decomposition approach whereby depolarization is modelled as a spatially localized "lump" phenomenon. In particular, the differential matrix methodology appears as particularly well adapted to the phenomenological description of the continuous scattering in turbid media.

In summary, the algebraic approaches to experimental Mueller matrices appear as a phenomenological methodology complementary to rigorous EM modelling. More generally, these represent very promising and efficient tools not to be neglected in our quest of physical comprehension of complex scattering phenomena.

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