

MUELLER MATRIX CHARACTERIZATION OF POROUS MEDIA IN VISIBLE

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ABSTRACT. In this paper, we apply Mueller polarimetry to study different samples of porous media compacted as tablets from a pharmaceutical excipient microcrystalline cellulose. We measured the Mueller matrices of the samples with the home made polarimeter using a He-Ne laser ($0.63 \mu\text{m}$). We show that polarization entropy manifests the highest sensitivity to the porosity allowing to identify the tablets of different porosities.

1. Introduction

In this paper, we introduce the Mueller matrix polarimetry for the purpose of characterization of scattering by porous medium. The samples of porous media are prepared in the form of tablets. The tablets (diameter 13 mm, thickness 3 mm and porosities 40.1% and 23.2%) were compacted from a pharmaceutical excipient, microcrystalline cellulose [(MCC) Avicell PH-200 FMC Biopolymer], with a compaction simulator (PU-UMAN PCS-1). More details on preparation of the tablets can be found elsewhere [1].

2. Experiment

In Figure 1 we present the geometry of experiment and the schematics of the polarimeter used in this study for the Mueller matrix measurements.

A detailed description of the polarimeter is given in [2]. Light from a linearly polarized continuous-wave *He-Ne* laser ($\lambda = 632.8 \text{ nm}$) passes through a polarization state generator PSG consisting of polarizer (*P*) and a wave plate (WP_1) with phase shift δ_1 and four azimuths α_1 . The light is subsequently scattered by a tablet sample. The scattered light passes through a polarization state analyzer PSA consisting of rotatable wave plate (WP_2) with phase shift δ_2 and an analyzer (*A*) and is detected by a detector, so that PSA is a complete Stokes polarimeter. Polarizer (*P*) and analyzer (*A*) are fixed and crossed relative to each other. The wave plates in PSG and PSA are assembled by holders controlled from the computer.

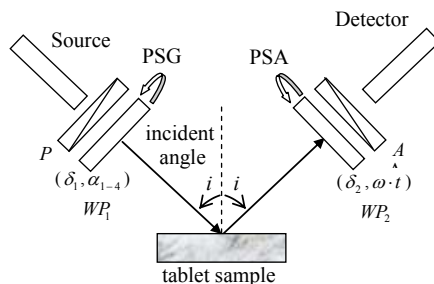


Figure 1. Schematic overview of the experimental geometry to measure the Mueller matrices.

3. Results

We measured the complete Mueller matrices for tablets with given porosities, i.e., sample 1 with porosity 40.1 % and sample 2 with porosity 23.2 %. The experimental geometry is presented in Fig. 1.

Figure 2 show the measured Mueller matrix elements for both tablet samples. Each point presented in figures below is a result of averaging over 500 realizations of the single measurements. Except for m_{11} all matrix elements are normalized to m_{11} , so that we consider m_{ij}/m_{11} , with $i, j = \overline{1, 4}$ aside from $i = j = 1$. There are no error bars shown in Fig. 2 because the values for the standard deviations are comparable with the symbols plotted and are below 2%. We investigated the reliability of the measured scattering matrices by checking that all of them satisfy the Cloude's test [3] within the experimental errors at each scattering angle. As it can be seen the eight matrix elements m_{13} , m_{14} , m_{23} , m_{24} , m_{31} , m_{32} , m_{41} and m_{42} are zero within the experimental errors over the entire incidence angle range and, thus, the Mueller matrix has a block-diagonal structure. So, from Fig. 2 we can deduce that most sensitive matrix elements for porosity identification are $m_{12} = m_{21}$, m_{22} and somewhat less sensitive m_{33} and m_{44} .

Further we investigated the sensitivity of Cloude's entropy [3, 4] for porosity identification. The results are presented in Fig. 3.

Cloude's coherency matrix can be derived from the Mueller matrix in the following way

$$H = \frac{1}{2} \sum_{j=1}^4 \sum_{i=1}^4 m_{ij} (\sigma_i \otimes \sigma_j^*) \quad (1)$$

where m_{ij} are the Mueller matrix elements; σ_i are the normalized Pauli matrices:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (2)$$

A requirement for a physically realizable Mueller matrix is that the coherency matrix H has non-negative eigenvalues [3]. For the average characterization of depolarization for given Mueller matrix the following single number depolarization metric- entropy - can be

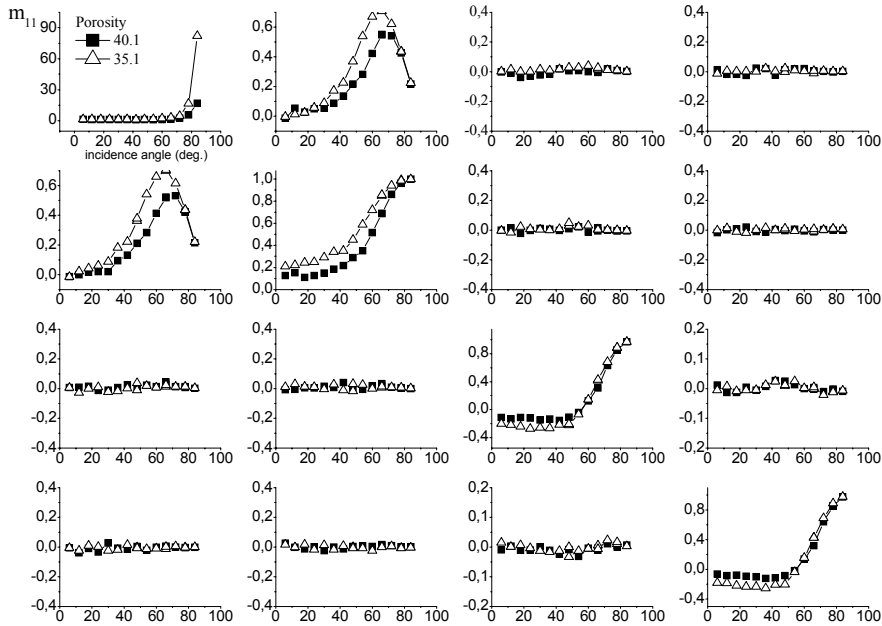


Figure 2. Mueller matrix elements for porous tablets.

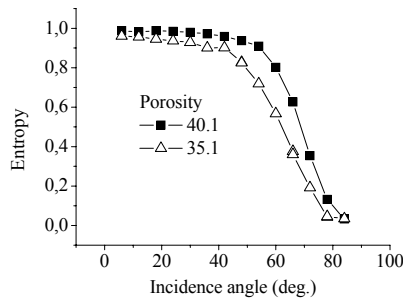


Figure 3. Cloude’s entropy for two tablet samples with porosity 40.1 % and 23.2 %.

used

$$H = \sum_{i=1}^4 -P_i \log_4 P_i \tag{3}$$

where $P_i = \lambda_i / (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$ and λ_i are the eigenvalues of H . For pure scattering without depolarization $H = 0$ and $\lambda_1 \neq 0, \lambda_{i \neq 1} = 0$. For totally depolarizing scatterers $H = 1$. When $H < 0.5$ and $H > 0.5$ one have weakly and strongly depolarizing cases, respectively.

Figure 3 shows that in the range of incidence angles $40^{\circ} - 80^{\circ}$ the entropy for samples studied differ validly. Thus, for these types of the samples we can state that at $\lambda = 632.8$ nm the sensitivity of Mueller polarimetry for porosity identification is considerably higher than 20 %.

4. Conclusions

In this paper, we present the results of the Mueller matrix measurements for estimation of porosity of two tablet samples, which were compacted from micro crystalline cellulose.

The patterns of measured matrix elements and the results of subsequent interpretation of the experimental Mueller matrices show that Mueller polarimetry at 632.8 nm enables us to identify the tablet porosity. The ratios m_{12}/m_{11} and m_{22}/m_{11} , Fig. 2, and the value of entropy for the incidence angle $\geq 40^{\circ}$, Fig. 3, give promise to identify the porosity of the tablet samples.

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