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LIGHT SCATTERING CALCULATIONS IN PLANAR NON-HOMOGENEOUS DIELECTRIC MEDIA BY MEANS OF THE LIGHT DIFFRACTION CALCULATION ON GRATINGS

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ABSTRACT. The generalized source method previously developed for the light diffraction calculation on periodic structures is applied for the light scattering calculation in non-periodic media. This greatly enlarges the domain of applicability of two-dimensional Fourier-methods in light scattering applications since the generalized source method has much less numerical complexity than widely used Fourier modal method. It is also demonstrated on numerical examples that for pure dielectric structures the use of lossy perfectly matching layers is not necessary for removing the effects of periodicity and taking of a large grating period is sufficient.

1. Introduction

Various models dealing with the light scattering on single particles and groups of particles usually consider scatterers placed in a homogeneous isotropic medium [1]. However there exists a range of problems where a volume with scattering particles is bounded by plane interfaces. For example one encounters such problems while optimizing organic solar cells and light emitting devices containing plane scattering layers being used for improving the devices external efficiencies.

Introduction of plane interfaces near ensembles of scattering particles was accomplished in the scope of several different approaches. For example, T-matrix calculations of the light scattering on a particle placed near a plane interface can be found in [2], however this approach may require non-trivial transformation between plane and spherical harmonics and might run into numerical instabilities. Regarding the finite difference time domain method as well as the finite element method, their important shortage recognized by many authors is high numerical complexity [1]. Volume integral methods intrinsically deal with finite-dimension scatterers and the introduction of infinite interfaces might be problematic.

In this work we propose an approach for modeling of the light scattering in plane inhomogeneous layers that uses the solution of a corresponding periodic problem. Taking a homogeneous layer is natural to treat the light propagation through it in terms of plane harmonics as they are eigensolutions of Maxwell equations for a plane homogeneous layer. This implies the operation in two-dimensional reciprocal space. In the diffraction grating theory the class of methods considering periodical modulations of plane layers in the reciprocal space is referred to as Fourier-based [3] and includes well-established and widely-used Fourier modal (FMM) and differential methods. Additionally, the generalized source method (GSM) [4] was recently shown [5, 6] to highly improve the capabilities of these Fourier-methods reducing the numerical complexity of the light diffraction calculations from $O(N_O^3)$ to $O(N_O)$ with N_O being the number of diffraction orders. This provides the possibility of considering quite complex dielectric permittivity distributions within the bounds of a grating period. Here we propose to treat an ensemble of scattering particles bounded inside a spatial volume as a period of an infinite grating. In previous works [7, 8] analogous attempts were made to suit the one-dimensional FMM for treating non-periodic problems by supplementing it with a perfectly matching layer (PML). Here we formulate the GSM in 2D reciprocal space with PMLs and demonstrate that for pure dielectric scatterers the use of PML is not necessary which simplifies considerably calculations.

2. Generalized source method in 2D reciprocal space

The GSM, which is applied here for the treatment of the light scattering problem in planar dielectric structures, represents a general approach for solution of the time-harmonic Maxwell equations [4]. In this work we apply it in two-dimensional reciprocal space of the diffraction grating k-vectors and one-dimensional space coordinate perpendicular to the grating plane. After discretization of the periodic structure into slices in the direction perpendicular to the layer plane and taking a finite number of Fourier harmonics the GSM results in a system of linear algebraic equations (close to that given in[5, 6]):

$$a = a_{inc} + TPU(M - QRPU)^{-1}Qa_{inc} \tag{1}$$

Here symbols a designate the amplitude vectors of TE and TM polarized plane harmonics; matrices U and M are composed of the Fourier harmonics of permittivities ϵ and μ and angular functions defining the scatterer surface geometry; matrices P and Q correspond to transitions between the field amplitudes and a; matrix R describes redistribution of diffracted harmonics in different slices; and matrix T – propagation of harmonics from slices to layer boundaries. The special feature of the system 1 is that all matrix factors that it contains are block-Toeplitz or block-diagonal. This gives the possibility of applying the Fast Fourier transform for performing matrix-vector multiplications, which together with the generalized minimal residual method results in the algorithm of linear complexity with respect to the product of the number of slices and harmonics [5, 6].

3. Non-periodic problem

The difference between periodic and non-periodic problems consists in two main features. First, outgoing waves diffracted on a periodic structure propagate along several certain directions defined by the structure period and the wavelength contrary to waves scattered on a single object that are continuously distributed over scattering angles. Second, each period of a periodic structure scatters not only incoming waves but also waves being scattered on all other parts of the structure. One can see from the above that the removal (or suppression) of interaction between different parts of a periodic structure may

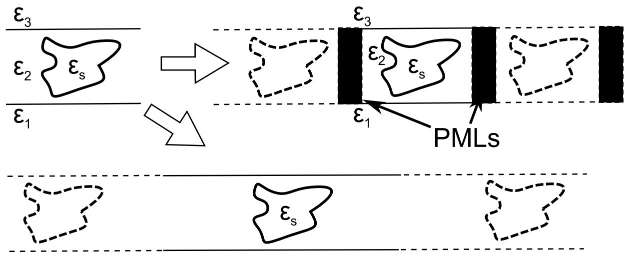

Figure 1. Two ways of transforming a non-periodic problem of the light scattering in planar media to a periodic problem of the light diffraction on gratings.

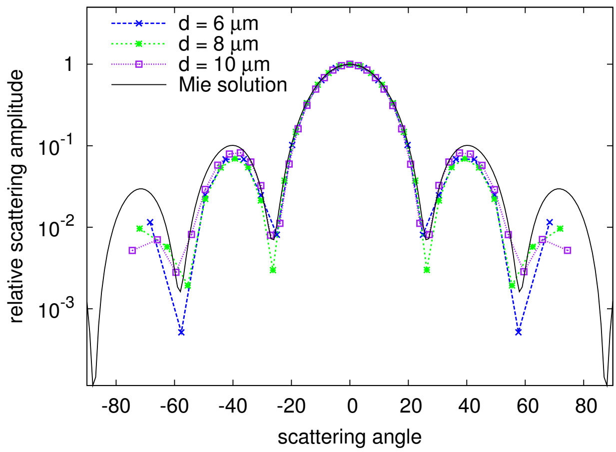

Figure 2. Comparison of the scattering diagrams obtained from the light diffraction calculation on a grating composed of spheres with the corresponding Mie solution.

lead to coincidence of scattering diagrams obtained for a single and for a periodized object in the corresponding directions providing that an appropriate normalization is used. With this purpose we incorporate the described model for light diffraction on gratings with PMLs inserted at the boundary of each grating period.

Contrary to widely used implementation of the PML as a complex coordinate transformation we treat it as a uniaxial medium which permittivities are related to basis values as follows:

$$\frac{\epsilon_o}{\epsilon_b} = \frac{\epsilon_b}{\epsilon_e} = \frac{\mu_o}{\mu_b} = \frac{\mu_b}{\mu_e} \tag{2}$$

where the indices 'o' and 'e' correspond to ordinary and extraordinary waves in the PML and the axis are perpendicular to the PMLs boundary to ensure that reflection coefficients for all propagating in a layer waves are brought to zero. Values ϵ_b and μ_b stand for the GSM basis permittivities.

Another way to suppress the electromagnetic interaction between different periods is to make the period sufficiently large in comparison with the size of a scattering object. To demonstrate this statement to occur we give a numerical example shown in Fig. 2. The relative scattering diagrams obtained from the light diffraction calculation on a periodized sphere are compared to the exact Mie solution describing the light scattering on the same single sphere. The sphere refractive index was taken to be 1.5 and the diameter to be approximately equal to the wavelength which was taken to be 0.5 μm. From the Fig. 2 it can be seen that for periods several times larger than the sphere size the scattering diagram obtained from diffraction calculations well reproduces the main features of the Mie solution – number of extremes, their positions and widths.

4. Conclusions

The generalized source method is implemented in two-dimensional reciprocal space and adjusted for the light diffraction calculation on gratings. It is supplemented with PMLs to perform light scattering calculations on non-periodic objects. Additionally, it was shown that the use of PMLs does not change results and, therefore, is not necessary. Light scattering calculation on dielectric bodies can be performed with immediate application of the grating model.

Acknowledgments

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