Multiobjective optimization of circuit performances through solution ranking and evolutionary strategies

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Abstract.

This work concerns the optimization of circuit performances of a Two Stage Operational Transconductance Amplifier. The problem arises from the circuit design flows which use simulators when these tools and their inner device models are considered as a black-box. The performance specifications are targets of the optimization and are explored in a multiobjective space. Solution ranking by Pareto dominance criterion leads to more stable multiobjective solutions which is an important consideration from the design point of view. Evolutionary strategies can exploit this solution rank by selection in order to get a set of non-dominated equivalent solutions called *Pareto optimal front*.

1. Introduction

In microelectronic design a considerably time is spent on device sizing of analog circuit in order to satisfy the performance requirements. The main reason is due to the non-linear relation between device sizes and performances.^{3,6}

In order to improve the efficiency of the device design in the analog circuits, multiobjective approach has been proposed as alternative to the approach of the cost functions. In this study the optimization process is coupled to a circuit simulator (SPICE) which evaluates the circuit performances.⁵ The Spice simulator version used in our experiments is *ngspice* (available at http://ngspice.sourceforge.net/), which implements the BSIM3 MOSFET model for the I-V characterization.^{1,4}

2. Two-Stage OTA Design

This case study proposes the MOS device sizing and the circuit net setting of an twostage Operational Transconductance Amplifier (OTA) (see figure 1). The OTA is a useful device and it is used with few other devices to realize filters, comparators, wave generator, converters, etc.

The circuit parameters and their ranges are showed in Table 1. The "W" parameters



refers to the MOS channel width, L is referred to the MOS channel length, R (resistance) and C (capacity) are referred to the circuit net parameters. Minimum performance specifications are formulated with the constraints in Table 3.

Parameters	Ranges	Unit
W1b = W1a	7 - 20	μm
W3	7 - 20	$\mu \mathrm{m}$
W5	7 - 20	$\mu \mathrm{m}$
L	0.525 - 0.875	$\mu \mathrm{m}$
С	3 - 5	pF
R	20 - 40	ΚΩ
W4	7 - 20	$\mu \mathrm{m}$
W2b = W2a	7 - 20	$\mu \mathrm{m}$
Ι	1 - 15	μA

Many important performance metrics are considered in the OTA design. Those used in this case study are the following:

Low frequency gain: It is the gain at 100 Hz, that is the base of the amplification range

Objectives	Specifications	Constraint	Unit
Power Consumption	minimize		Watt
Total Width	minimize		μm^2
Unity Gain Frequency	maximize	> 31.221	Mhz
Gain at 100 Hz	maximize	> 64.118	dB
Phase Margin	maximize	> 60	Degree

- **Unity Gain Frequency:** It is defined as the frequency range where the amplifier has at least the unity gain
- **Phase Margin:** It is an indirect quality measure for the circuit because it is related to the parasitics effects, like cross coupling, which causes the failure in the attainment of the performances.
- **Circuit's Area:** This metric is related to the yield of the manufacturing process. When the number of circuit per unit area increase then the yield of the manufacturing process increase. In our case we used an underestimation given by the sum of the MOS widths.
- **Power Consumption:** It is today an important performance for all systems in which the power is supplied by a battery.

3. Evolution strategies for scalar optimization

Usually, an evolution strategy is initialized with a population of random feasible entities individuals also named chromosomes which represent potential solutions to the given optimization problem. These individuals are reproduced such that attributes from different parents can be given to an offspring by recombination or cross-over operators. The mutation operator can include random errors during reproduction. For each individual is defined a fitness function which possibly depends on the environment. Each individual of the offspring population is evaluated according to its fitness function such that only better ones are selected as parents of the next generation, there is an elitist selection when some of the better individual from the current generation are carried over to the next generation unaltered. This process is iterated until a stopping criterion is fulfilled, e.g. a maximum number of generations.

The *i*-th individual of generation *t* can be written as an *n*-dimensional vector a^{it} with components $a_1^{it}, \ldots, a_n^{it}$ representing an alternative as a point in \mathbb{R}^n . Possibly additional components $a_k^{it}, k > n$, are used for storing control information of the evolutionary process and they are called strategic components. The $(\mu + \lambda)$ -evolution strategy, where $\mu, \lambda \in N$, starts with a population (t = 0) of μ feasible parents $a^{i0} \in A$ which produce λ offspring. During the reproduction, mutations occur as $(0, \sigma^t)$ -normally distributed vector-valued random variables $z^{it} \in \mathbb{R}^n$, such that offspring $a^{it+1}, i \in \{1, \ldots, \lambda\}$ is calculated as:

$$a^{it+1} = a^{jt} + z^{it}$$

for $j \in \{1, ..., \mu\}$.

For each offspring the fitness function f is evaluated. If a restriction g is violated the fitness can be modified using a penalty function. Alternatively only feasible mutations

 $a^{it+1} \in A$ are allowed. This is especially important for the comma-evolution strategy (μ, λ) -ES with $\lambda > \mu$ where parents live one generation only. Here, possibly more than λ offspring have to be generated to ensure a constant population size of μ feasible alternatives.⁸ The μ best of the offspring become parents of the next generation in t + 1. With the alternative $(\mu + \lambda)$ -evolution strategy, the live span of parents is not limited. In the selection step, offspring and parents are considered such that parents can survive several generations if they are fitter than their offspring. This also prevents a temporary deterioration of population fitness.

The distribution parameter $\{\sigma^t = (\sigma_1^t, \dots, \sigma_n^t) \in \mathbb{R}^n\}$ for $i \in \{1, \dots, n\}$, for the mutations can be interpreted as a step size vector analogously to deterministic search strategies. Based on theoretical considerations, Rechenberg (1973) proposes a 1/5 success rule further specified by Schwefel. This rule is based on an increase of the step sizes if on average the portion of successful offspring (i.e. with increased fitness) is larger than 1/5. If the portion is less than 1/5 the step sizes are decreased. This step size control does not support direction-specific adaptations. It is only possible to prescribe constant scaling factors for the co-ordinate directions because the σ_i remain in constant proportions (as long as they do not reach a minimal value > 0). The 1/5 rule fails when there are no continuous partial first derivatives of the objective function. Because of these problems, Schwefel⁸ proposes another, more natural concept of step size control which allows an automatic scaling of the variables: the step size parameters are themselves controlled evolutionarily by adding n step size parameters to the n alternative parameters. Both types of entity parameters are mutated by normally distributed random variables. The step sizes are then controlled indirectly by the selection mechanism with an unchanged fitness function.

Schwefel also discusses some other mutation concepts which, for instance, allow a learning of search directions independently from the co-ordinate axes. Another important mechanism in evolution strategies introduced by Schwefel is recombination. This simulation of sexual reproduction is based on the idea that the genetic material of an offspring does not come from a single parent but from two in nature. Schwefel proposes to choose each component of an offspring vector randomly with an equal probability of $1/\mu$ from the parent population. Recombination can also be used for the control parameters of an individual. Because of stability reasons intermediary recombination is proposed such that the mean value of two parents is inherited to the offspring.

4. Evolutionary Algorithm MultiOb

Evolutionary Algorithm MultiOb was developed by Fraunhofer Institut Techno- und Wirtschaftsmatematik (ITWM).² This algorithm was modified in order to optimize integrated circuit performances.

The bounded multiobjective problem, optimized by MultiOb, is expressed by this formalization:

(1)
$$\min_{\mathbf{x} \in H \subset B^n} \mathbf{f}(\mathbf{x})$$

with $f_i : \mathbb{R}^n \leftarrow \mathbb{R}^q$ continuous functions. In this formulation, n is the number of parameters (decision variables), q is the number of objective functions, **f** is the vector-valued

objective function, H is an hyperrectangle in the parameter space. The objective functions can be also assumed to be Lipschitz continuous. Optimization is done according to a black box concept: optimization algorithm takes no advantages of knowing an explicit formulation of the objective functions. A technique from the family of robust metaheuristics has been chosen, it supports the generation of (approximate) solutions according to the Pareto dominance criterion.

The algorithm uses a number of parameters such as the population size (for parents and offspring), the executed number of generations, and settings of the applied evolutionary operators such as an average step size for mutations, probability of recombinations between solutions or probability of recombination between each component of solutions. In general, the settings of these parameters influence the performance of the EA and the computational effort. In brief, approximation quality improves with the population size and the number of generations while the computation effort grows linearly with each of these parameters.

The selection criterion chooses the individuals in accordance with the Paretodominance.

Definition 4.1 (Pareto dominance). Given $\mathbf{y}', \mathbf{y}'' \in \mathbb{R}^n$, \mathbf{y}' dominates \mathbf{y}'' if

(2) $(\forall 1 \le i \le m : y'_i \le y''_i) \land (\exists j : y'_j < y''_j)$

Algorithm 4.1 Pseudo code of MultiOb algorithm
Require: population_size ,population_offspring, generation_number
1: Initialize a random population of <i>population_size</i> individuals
2: Evaluate the objective functions on the population
3: for $i \leftarrow 0$ to generation_number do
4: Generate a population of dimension <i>population_offspring</i> by mutation and recom-
bination operators
5: Evaluate the objective function on the new population
6: Select by Pareto Dominance criterion
7: Update the mutation rate
8: end for
9: Save results and make statistics
This kind of objective space optimization can lead to set of stable solutions in mul-
tiobjective sense. ⁷ This characterization leads to a robust search and give back high

tiobjective sense.⁷ This characterization leads to a robust search and give back high quality design. The algorithm also implements elitist strategies where good parents are saved from selection. This strategies is used to improve the convergence. As a result, the routine(s) will return sets of equivalent solutions according to the Pareto Relation, i.e. approximations of the efficient set. Algorithm 4 shows the pseudocode of MultiOb.

5. Results

The MultiOb Algorithm was set with the configuration parameters showed in table 1.

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Evolutionary Parameter	Value
Population size (parents)	1000
No. of offspring	500
No. of generations	5000
Mutation rate	0.1
Recombination rate 1	0.1
$Recombination \ rate \ 2$	0.25



Fig. 1. Trade-off among Phase margin, Gain at 100 Hz and Unity Gain Frequency

The figures 1 and 2 show the trade-off among triples of objectives.

In figure 3 is showed the tradeoff between power consumption and unity gain frequency from MultiOb sampling. Notice that a small set of representative points are selected by the algorithm. These points have the highest dominance with respect the Pareto criterion and they are stable in a multiobjective sense.⁷

6. Conclusion

In this case study a circuit design was carried out by multiobjective optimization. Multiobjective characterization has located stable solutions. These solution can lead to more quality design because quantifies tradeoffs for multiple competing goals in circuit design. The plurality of design point can represent a set of alternatives useful for the synthesis of electronic systems.



Fig. 2. Trade-off among Power dissipation, Gain at 100 Hz and Unity Gain Frequecy

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Fig. 3. Trade-off between Power dissipation and Unity Gain Frequency