

# MODELLING CHANGES OF HYDRAULIC PROPERTIES INDUCED BY BIOMASS GROWTH IN UNSATURATED POROUS MEDIA

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## 1 Introduction

The effect of the microbial growth on hydraulic properties of porous media is a topic studied in the framework of many applications, such as oil recovery, water and wastewater treatment, bioremediation of contaminated soils.

In particular, we shall focus on the biodegradation processes, where various interactions between physical and chemical phenomena have to be taken into account in order to get useful and realistic models.

The bioremediation of soils and aquifer systems has been deeply studied, also from a mathematical point of view (see, for instance, [1] and the reference therein).

The studies on the effects of biomass on porous media flow are presented in the papers by Rockhold et al., [2], [3], [4]. The authors give an accurate description of the experimental results concerning the bacterial-induced changes of the medium properties, such as density, contact angle, surface tension, porosity, etc.

As stated in the cited papers, additional work is needed in modelling unsaturated condition, while the saturated case has been deeply investigated (see, for instance, the paper by Thullner et al. [5], where homogenization techniques are used).

Concerning the literature of unsaturated condition, in [6] and [7] the properties of absorbent porous media are studied, along with the processes of granular growth and porosity variation. Although such works have given fundamental results, they do not treat the case of permeability directly dependent on the saturation degree (which is time varying).

The objective of this study is thus to analyze the flow through unsaturated porous media (possibly contaminated by several chemical species) in presence of biomass growth (resorption) processes which induce changes in the hydraulic properties of the medium itself.

Notice that, a rigorous approach should be the one presented in [5], generalized to the unsaturated case. Nevertheless, such techniques are very involved and their implementation seems to be too far for models devoted to applied sciences.

## 2 Problem description

We consider a “column experiment” constituted by a sample of a contaminated unsaturated soil, which represents a “laboratory scale” of a real vadose zone.

We call  $x$  the vertical coordinate of the column, pointing upwards.

At the initial time ( $t = 0$ ) the saturation degree of the medium is considered in a steady state, as well as the pollutant concentration therein. Our goal is to model the evolution of the system after a certain quantity of biomass is inoculated into the column.

Hereafter we list the most significant physical assumptions on which the model is based.

- (P.1) The soil is taken as a homogeneous, rigid porous medium considered as a network of capillary tubes, uniformly distributed.
- (P.2) The pollutant present in the column is dissolved in water and adsorbed onto the soil grains.
- (P.3) The biomass within the medium is distributed in water as suspension (let us call this fraction “*free biomass*”) or attach on the soil grains (“*attached biomass*”). In particular:
- There is no cluster formation in the free biomass, namely the microbial particles do not agglomerate. The volume occupied in water by the bacteria is not significant. Such an assumption allows to model the mixture (water and free biomass) as a fluid whose physical characteristics are changing in time.
  - The attached biomass forms a biofilm, whose thickness varies in time, completely covering the internal surfaces of the soil pores. The biofilm is a porous medium as well so that the (contaminated) water can diffuse through it reaching the pores surface. The biofilm has a constant porosity and is saturated at all times. Therefore, it is considered as a (incompressible) mixture of solid biomass and immobile water.
- (P.4) We consider the attachment of free biomass on the biofilm, but we neglect the inverse process (i.e. we neglect detachment). Indeed the experiments show that the detachment process is mainly due to the mechanical action caused by the “fast” water flux, [8]. We, indeed, consider pretty “slow” fluxes, mainly due to the porosity and permeability variations caused by the microbial growth. We therefore neglect the mechanical influence of the water flux on the biofilm formation and growth.

Free and attached biomass are responsible for different effects changing hydraulic properties. More precisely:

(V.1) The free biomass causes viscosity, density and surface tension variations.

(V.2) The biofilm growth (resorption) causes medium porosity variations and affects the contact angle.

The above variations induce, in turn, changes in the permeability and in the relative saturation of the medium.

REMARK 1. *Even if we are assuming that the fluid characteristics change in time, we shall neglect the fluid density variation. This may be significant only in case of total saturation.*

### 3 The model

We now give the basic definition and then simply list the equations that will be used. We first introduce the following quantities:

- $\phi_s$ , volume fraction occupied by the solid matrix (constant and uniform),  $[\phi_s] = [-]$ . In particular, we introduce  $\varepsilon_0 = 1 - \phi_s$ , as the initial porosity of the column, which is known.
- $\phi_f$ , volume fraction occupied by the fluid (mixture of water and air),  $[\phi_f] = [-]$ .
- $\phi_b$ , volume fraction occupied by the biofilm (mixture of biomass and immobile water),  $[\phi_b] = [-]$ . We introduce also the biofilm porosity,  $\varepsilon_b$ , which is considered a known constant.
- $\sigma$ , water saturation,  $[\sigma] = [-]$ .
- $\theta_{lm} = \sigma\phi_f = \frac{\text{volume of "mobile" water}}{\text{porous medium volume}}$ ,  $[\theta_{lm}] = [-]$ .
- $\theta_{lb} = \varepsilon_b\phi_b = \frac{\text{volume of "biofilm-stored" water}}{\text{porous medium volume}}$ ,  $[\theta_{lb}] = [-]$ .

We thus have that

$$(3.1) \quad \phi_f + \phi_b = \varepsilon_0,$$

and that the volume fraction occupied by the liquid (accounting for mobile water and stored water) is

$$(3.2) \quad \phi_l = \theta_{lm} + \theta_{lb} = \sigma\phi_f + \varepsilon_b\phi_b.$$

We now list all the dependent variables which have to be determined by the problem solving, namely:

- $\sigma(x, t)$ , the saturation of mobile water.
- $\phi_b(x, t)$  (or, alliteratively,  $\phi_f(x, t) = \varepsilon_0 - \phi_b$ ).
- $N_l(x, t) = \frac{\text{number free bacteria in mobile water}}{\text{unit mass of mobile water}}$ ,  $[N_l] = Kg^{-1}$ .

- $w_A(x, t) = \frac{\text{mass of pollutant soluted in water}}{\text{unit mass of water}}$ , i.e. the pollutant concentration in water  $[w_A] = (p.p.m.)$ .
- $w_s(x, t) = \frac{\text{mass of pollutant adsorbed onto soil grains}}{\text{unit mass of solid matrix}}$ , i.e. the concentration of the adsorbed pollutant  $[w_s] = (p.p.m.)$ .

We assume to know, along with  $\varepsilon_b$  and  $\varepsilon_o$ ,  $N^*$  the number of bacteria that form the unit volume of biofilm.

Concerning the equations, we just list them without giving a detailed description:

- $\frac{\partial}{\partial t} (\theta_{lm} + \theta_{lb}) = -\frac{\partial q}{\partial x}$ , Richards equation.
- $\frac{\partial}{\partial t} (N_l \theta_{lm}) = -\frac{\partial}{\partial x} (q N_l) + \frac{\partial}{\partial x} (a_L q \frac{\partial N_l}{\partial x}) + \Gamma_{N_l}$ .
- $\frac{\partial \phi_b}{\partial t} = \Gamma_{\phi_b}$ .
- $\frac{\partial w_s}{\partial t} = \Gamma_{w_s}$ .
- $\frac{\partial}{\partial t} (w_A (\theta_{lm} + \theta_{lb})) = -\frac{\partial}{\partial x} (q w_A) + \frac{\partial}{\partial x} (a_L q \frac{\partial w_A}{\partial x}) + \Gamma_{w_A}$ .

Starting from an investigation on steady state, we want to simulate the influence of the biomass growth on the water redistribution within the soil sample.

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