

## A FE simulator for electromagnetic analysis of slow-wave helicoidal structures in Traveling Wave Tubes

S. COCO, A. LAUDANI\* and G. POLLICINO

*DIEES, University of Catania,  
Catania, I-95125, Italy*

*\*E-mail: alaudani@diees.unict.it*

R. DIONISIO and R. MARTORANA

*Galileo Avionica  
Palermo, I-90125, Italy*

### Abstract.

In this paper the authors present a Finite Element tool expressly conceived for the analysis of TWT helicoidal slow wave structure (SWS). The SWS is the region of interaction between the beam and the RF signal. Accurate analysis of the SWS is a difficult task due to its complex geometry. Moreover in order to achieve the design parameters a specific post-processing is required. Consequently specialized 3D numerical simulators are needed to carry out the design process. The FE tool expressly conceived to treat this kind of analysis includes a dedicated mesh generator, an electromagnetic solver, and a post-processing module. Analysis of typical helical SWS are also illustrated and discussed.

*Keywords:* Eigenvalues and eigenfunctions; Finite Elements Method; Slow wave structures; Traveling Wave Tubes

### 1. Introduction

Traveling Wave Tube (TWT) are electronic vacuum devices used for high-power amplification of RF signals.<sup>2,3</sup> Three main regions are usually individuated within an TWT: the electron gun, where the beam is generated, the slow wave structure (the interaction region), where the electron beam exchanges its energy with the RF signal and a depressed collector, where the beam energy is recovered. The slow wave structure (SWS) is a periodic waveguide, usually constituted by an helix or a series of coupled cavities. A critical step of TWTs design is the electromagnetic analysis of its SWS, including calculation of the SWS dispersion, interaction impedance and RF losses. In order to compute these parameters it is possible to set up a so-called cold test, an electromagnetic analysis performed without the electron beam, for which experimental measures are easily made. Only in recent years computer simulation of the cold-test parameters has been used for some TWT SWSs.<sup>1</sup> Nevertheless accurate simulation of the helicoidal SWSs is a difficult task due to its complex geometry. Furthermore in order to achieve the useful design parameters a specific post-processing is required. Consequently specialized 3-D numerical simulators, appositely tailored to meet the above requirements, are needed to

carry out the analysis of helicoidal SWS. Currently available tools based on the finite difference method (FDM) or Finite Integration Technique (FIT) may not be adequate for this purpose since they do not allow a very flexible meshing. By pursuing a finite-element (FE) approach, the above limitations can be overcome thanks to mesh refinement and adaptive mesh generation in critical regions. In this paper the authors present a Finite Element tool expressly conceived for the analysis of TWT helicoidal SWSs. The paper is structured as follows: in the section II a brief description of the simulator is presented; in section III the FE analysis of helicoidal SWSs and the construction of Brillouin diagram are illustrated; in section IV an example of analysis of a typical helicoidal SWS is described.

## 2. Description of the simulator

The simulator is written in the C++ language and has been developed under the MS-WINDOWS OS environment. Its structure follows the classical scheme of FE simulators in which three main modules are present. The first one groups pre-processing functions (such as the construction of the geometrical and functional model and FE mesh generation). The second module is devoted to the processing functions (solution of the mathematical model). The last module is devoted to post-processing (such as further elaboration of results, analysis plots, etc.). The tool has been specifically conceived to provide the TWT designer with an easy-to-use environment through a friendly CAD-based Graphical User Interface (GUI). Differently from other interfaces dedicated only to post-processing, it aims at facilitating the management of all the various aspects of a simulation session, allowing the interactive execution of all the simulator functions. The GUI also includes visualization tools, together with some specific pre/post-processing functions related to TWT design and global parameters extraction, including the automatic building of the Brillouin diagram, from which the phase velocity is evaluated. In the following we discuss more in detail the processing function and the building of Brillouin diagram.

## 3. FE Analysis of helicoidal SWSs and Brillouin diagram construction

In order to evaluate the performance of the interaction region of a TWT the construction of the Brillouin diagram plays a fundamental role, allowing the designer to study the variation of the phase and group velocity as a function of the frequency. Consequently in the following we concisely illustrate this diagram and its construction. In the electromagnetic analysis of a periodic structure we consider the Floquet's theorem applied to the solution of Maxwell equation. According to this theorem the electric (or magnetic) field of a periodic structure of period  $L$  and direction of propagation  $z$  can be expressed in the following way:

$$(1) \quad E(x, y, z - L) = E(x, y, z) \exp(j\beta_0 L)$$

where  $\beta_0$  is a complex number. Thus the solution of the Maxwell equations can be written:

$$(2) \quad E(x, y, z - L) = E_p(x, y, z) \exp(j\beta_0 z)$$

where  $E_p(x, y, z)$  is a periodic function of period  $L$ , which using the Fourier expansion becomes:

$$(3) \quad E_p(x, y, z) = \sum_{n=-\infty}^{+\infty} E_n(x, y) \exp(-j\frac{2\pi n}{L} z)$$

Assuming

$$(4) \quad \beta = \beta_0 + \frac{2\pi n}{L}$$

the equation 3 becomes

$$(5) \quad E_p(x, y, z) = \sum_{n=-\infty}^{+\infty} E_n(x, y) \exp(-j\frac{2\pi n}{L}z)$$

The vectorial quantities  $E_n(x, y)$  are called spatial harmonics. Consequently in a periodic structure a propagation mode consists of an infinite number of spatial harmonics, having different phase velocity in the propagation direction  $z$  given by:

$$(6) \quad v_{pn} = \frac{\omega}{\beta}$$

Therefore the propagation mode at a given frequency is not characterized by an exclusive phase velocity as for uniform waveguides. The set of curves  $\omega - \beta$  is called Brillouin Diagram. The construction of such a chart can be simplified considering the periodicity of  $\beta$  given by equ. 4. For an helicoidal SWS the electromagnetic analysis is a very complicated task and analytical solutions are not available in closed form. Usually an approximated model for this kind of SWSs can be considered, that is the sheath helix, a mathematical model consisting of a continuous cylindrical sheath of current flowing around the cylinder at a pitch angle.<sup>2</sup> In this way it is possible to trace an approximated Brillouin Diagram, which is shown in fig. 1. In this case  $a$  is the average radius of the helix,  $\psi$  is the pitch angle (that is  $\tan \psi = \frac{L}{2\pi a}$ ).

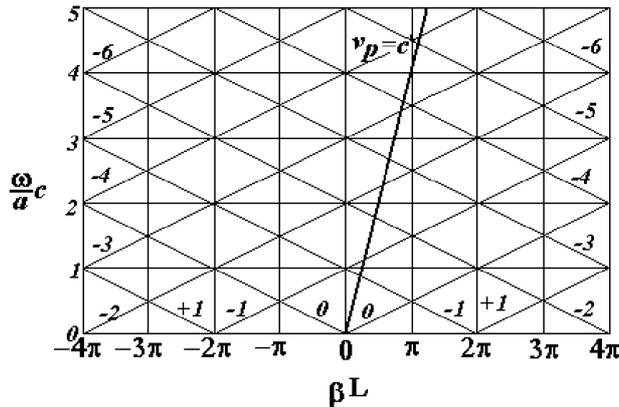


Fig. 1. Brillouin diagram for a sheath helix ( $\tan \psi = 0.125$ )

The helicoidal SWS is a cylindrical waveguide containing a tape helix and some helix supports, called rods. From the electromagnetic point of view such a structure is a periodic

waveguide and consequently the FE analysis must take into account the complete 3-D discretization of a spatial period of the helix. The electromagnetic problem is governed by the vector Helmholtz wave equation for the electric field  $\mathbf{E}$  in its residual form:<sup>5</sup>

$$(7) \quad \int_V (W) \cdot \left( \nabla \times \left( \frac{1}{\mu_r} \nabla \times E \right) - k_0^2 \epsilon_r E \right) dV = 0$$

where  $k_0^2 = \omega^2 \mu_0 \epsilon_0$  and  $V$  is a volume containing a period  $L$  of the helix. In order to avoid spurious modes, edge elements are used to express the electric field and weights  $W$ . Assuming the Floquet boundary for the boundary surface  $\mathbf{Sp1}$  ( $z = 0$ ) and  $\mathbf{Sp2}$  ( $z = L$ ) (periodicity along  $z$ -axis is assumed)

$$(8) \quad E(x, y, z + L) = E(x, y, z) \exp(-j\beta_0 L)$$

and imposing the boundary conditions to the equ. 7 we obtain the following eigenvalue problem:

$$(9) \quad [S][e] = k_0^2 [T][e]$$

where the matrices  $[S]$  and  $[T]$  can be built from the respective local matrices  $[S_E]$  and  $[T_E]$  for the element  $E$ , whose entries are:

$$(10) \quad s_{ij} = \int_E \nabla \times \tau_i \cdot \frac{1}{\mu_r} \cdot \nabla \times \tau_j dE$$

$$(11) \quad T_{ij} = \int_E \tau_i \cdot \epsilon_r \cdot \tau_j dE$$

In the construction of the mesh for the considered domain the elements can be conveniently distinguished into three different classes: those belonging to the surface  $\mathbf{Sp1}$  (class 1), those belonging to the surface  $\mathbf{Sp2}$  (class 3), and the remaining elements (class 2). In the following we consider the vector  $[e]$  of the edge elements subdivided into three parts, according to the class to which the edge belongs. If the mesh is built discretizing in the same way the two periodic surfaces we have  $e_3 = e_1 e^{j\beta L}$ , and consequently the eigenvalue equation 9 assumes the following form:

$$(12) \quad [A(\beta)][e] = k_0^2 [B(\beta)][e]$$

where the matrices  $[A(\beta)]$  and  $[B(\beta)]$  depend on the coefficient  $\beta$ , ranging in the interval  $[-\frac{\pi}{L}, \frac{\pi}{L}]$ . In the case of homogeneous and lossless material the two matrices  $[A(\beta)]$  and  $[B(\beta)]$  for an assigned propagation parameter  $\beta$  are hermitian and can be used to obtain eigenvalues and eigenvectors. A Jacoby-Davinson method has been adopted for the solution of such eigenvalue problem, providing the target eigenvalue range by means of the Lanczos method.<sup>4</sup>

#### 4. An example of analysis of a typical SWS

The analysis of the helicoidal SWS shown in fig. 1 is presented hereafter. In particular the fig. 1 shows the discretization of the helix generated by the appositely developed mesh generator and visualized by the postprocessor module. The eigenmode analysis has been performed for this structure in order to build the Brillouin diagram. In fig. 2 the first five modes of the SWS computed by the developed tool are plotted for different values of  $\beta$ . The analysis has been repeated several times using different finite element discretizations in order to assess its sensitivity to the number and kind of finite elements employed. The results obtained for the various analyses showed negligible differences.

## 5. Conclusions

The 3-D FE simulator developed is a dedicated tool for the electromagnetic analysis of SWSs in TWTs. Its main advantages are its capability to discretise and analyse accurately realistic device structures and the possibility to provide the user with all the interesting design parameters thanks to its powerful postprocessing.

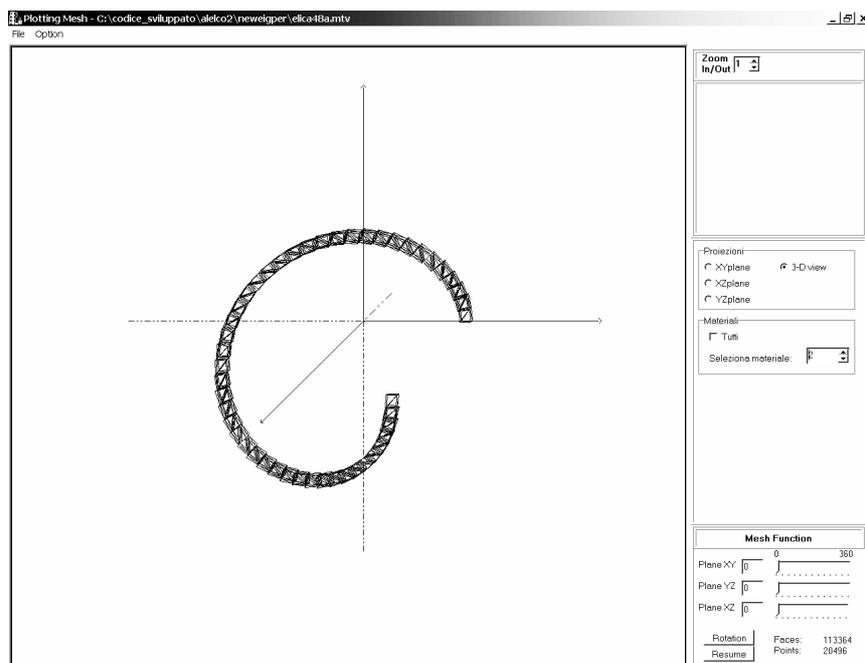


Fig. 1. 3-D Visualization of the helix discretization

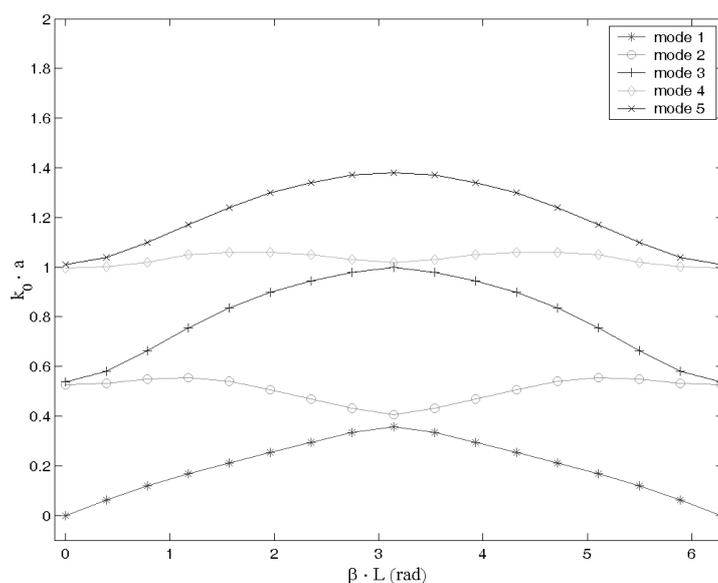


Fig. 2. The dispersion plot of the first five modes

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