

SPH Method Applied to Naval-Hydrodynamic Problems

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Introduction. Common problems in naval hydrodynamic and coastal engineering are the studies of general internal and external flows. Classic methods of solution have to face break down when dealing with large deformations and fragmentations of the air-water interface. Possible algorithms of solutions can be based on fixed-grid solvers of the fluid dynamic equations coupled with techniques to capture the interface evolution, like *Level Set* (LS) or *Volume Of Fluid* (VOF). These methods proved to be suitable in many circumstances but still much work is required to improve them in terms of solution validity. An alternative *meshless* technique is the *Smoothed Particle Hydrodynamics* (SPH) method. For it, no computational grid is introduced in the domain (meshless character) and the flow evolution is described following the motion of a set of fluid particles (Lagrangian character). It has been applied to the study of some internal (sloshing and dam-break problem) and external flows (breaking and post-breaking evolution of bores propagation toward beaches and bow breaking waves generated by fast slender vessels).

Governing equations. The SPH formulation assumes the fluid inviscid and the flow free to have rotational motion; so that the problem is governed by the Euler equation. Moreover, the SPH method considers the flow as weakly-compressible, meaning that the speed of sound $c = \sqrt{dp/d\rho}$ is at least one order of magnitude greater than the maximum flow velocity. Since the flow is assumed isentropic, two additional equations are then required to complete the problem: a continuity equation for the density field ρ , and an equation of state directly linking the latter quantity to the pressure field. The resulting system of PDE is:

$$(0.1) \quad \frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \mathbf{g}; \quad \frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}; \quad p = \frac{\rho_0 c_0^2}{\gamma} \left[\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right]$$

where c_0 is the speed of sound evaluated in absence of compression, *i.e.* with $\rho = \rho_0$. In the Tait state equation of (0.1), classically chosen to model water, the value $\gamma = 7$ is used [1].

SPH Numerical Scheme. In meshless methods, the field of a generic quantity f is represented through convolution integrals over the domain Ω $\hat{f}(\mathbf{x}) = \int_{\Omega} f(\mathbf{x}^*) W(\mathbf{x} - \mathbf{x}^*; h) dV^*$ in which $W(\mathbf{x} - \mathbf{x}^*; h)$ is a weight function and h a characteristic length of its bounded support Ω_{x^*} . The latter is defined as the area where W differs from zero. Physically, h is also representative of the domain of influence Ω_{x^*} of \mathbf{x}^* . The weight function W , called *smoothing function* or *kernel* in the SPH framework, is positive, centered in \mathbf{x}^* and decreases monotonously with $\|\mathbf{x} - \mathbf{x}^*\|$ to reach zero at the border of its support Ω_{x^*} . Its integral on this support is unity. When calculation the limit as $h \rightarrow 0$, the kernel function W becomes a Dirac delta function, and thus \hat{f} turns to be exactly f .

In particle methods, the fluid domain Ω is discretized as a finite number N of *particles* which represent small volumes of fluid dV , each one with its own local mass and other

physical properties. From the discrete form of the integral interpolation the problem governing equations (0.1) are written as:

$$(0.2) \quad \begin{aligned} \left[\frac{D\mathbf{x}}{Dt} \right]_i &= \mathbf{u}_i & \left[\frac{D\rho}{Dt} \right]_i &= -\rho_i \sum_j (\mathbf{u}_j - \mathbf{u}_i) \cdot \nabla_i W_j(\mathbf{x}_i) dV_j \\ \left[\frac{D\mathbf{u}}{Dt} \right]_i &= -\frac{\sum_j (p_j + p_i) \nabla_i W_j(\mathbf{x}_i) dV_j}{\rho_i} \end{aligned}$$

as described in [2]. To let the problem (0.2) evolve in time, modified-Euler temporal scheme is used. To further improve the stability properties and, in general, the performances of the solver, some modifications to the basic scheme previously described are introduced in [2].

Analysis of some Marine Hydrodynamic Problems: External Flows. Ship-generated waves have always fascinated scientists, and played a key role in surface-ship hydrodynamics contributing to hull resistance, generating noise and radiating very long narrow wakes remotely visible. Some of these phenomena originate abeam the ship in the form of extensive breaking of diverging bow and stern waves, eventually developing in wakes. An example from real life is shown on the left of figure 0.1 for a yacht during an American's Cup competition. From the picture the main features of the breaking bow waves and the related wake can be detected. In the following the complex fluid dynamics involved in the bow-wave radiation, including wave breaking, is discussed by means of the results of SPH calculations. The analysis is limited to high speed slender ships, with a sharp stem, for which basic insights can be achieved by an approximate quasi three-dimensional model based on the idea that longitudinal gradients of relevant flow quantities are small compared with vertical and transverse gradients. This approximation is called 2D+t. The 3D problem is considered equivalent to the unsteady two-dimensional free-surface flow generated by a deformable body in the vertical plane transverse to the ship. This deformable body coincides with the ship cross section in that plane which deforms as the ship moves forward (see sketch in figure 0.2). Within the 2D+t framework, the nonlinearities induced by hull and free-surface deformations are fully retained. On the right of figure 0.1 the three-dimensional reconstruction of the wave pattern generated by the vessel advancing with a Froude number $Fr = 0.41$ is shown. This result has been obtained through the SPH calculation. The presence of a bow flare induces the waves generated by the vessel to break sooner and closer to the hull. For the used Froude number, the water run-up along the ship bow sides causes a quick formation of a plunging wave breaking very close to the ship, see plunge point indicated with letter A on the right of figure 0.1. The water-water impact initiates a cyclic process of splash up. In this case, two splash-up events are observed, B and E. The further breaking of each splash-up implies the formation of vorticity for topological reasons. Each vortical structure is left behind by the breaking front and in a 3D perspective and results in a wake proceeding quasi parallel to the ship longitudinal axis. Two longitudinal wakes, G, occur in the studied case and are indicated by the two arrows in the figure. The wakes leave a scar, D, on the free surface which characterizes in a way the specific ship bow. Between the wakes and the hull a quite flat region forms, a sort of smooth plateau, C, where the free surface is practically undisturbed by the ship forward motion. In a flow region quite advanced with respect to the fore bow a quasi-stationary spilling breaker, F, remains as memory effect of the phenomenological process described.

Analysis of some Marine Hydrodynamic Problems: Internal Flows. Besides the flow around the ship, it is very common that some fluid is present inside the vessel. For example there can be water shipped on to the decks through the green-water events or there can be fluid stored inside the tank as for the LNG (Liquid Natural Gas) carriers. Here the problem of the motion of the fluid inside moving tanks, commonly referred to as *sloshing*, will be described through the numerical method presented above. The simplified problem of a two-dimensional square tank, with fluid depth h and tank breadth L , is analyzed and the features of two sloshing phenomena are described. A sinusoidal horizontal oscillation is imposed to the reservoir of water which is characterized by a finite depth $h = 0.35L$, (see figure, 0.3). As an example, in the present work, we describe a sloshing flow characterized by large motion of the fluid with fragmentation of the free-surface. This is obtained using an excitation period $T = 0.945T_1$, (where T_1 is the first natural period), and an amplitude $A = 0.05L$ for the horizontal motion. A linear theory would foresee a wave height modulated with the difference between the frequency imposed and the first natural frequency, and with a maximum value proportional to the amplitude of oscillation [3]. Successively the viscosity damps out the modulation leaving the free-surface oscillating with the period of excitation. In the experiments (see [4]) non linear phenomena characterize the sloshing flow and free-surface fragmentations alternate at the two sides of the tank with a period different from T . Numerical simulations have been used to understand the reason of such differences between linear theory and practical applications. In figure 0.3 the vorticity contours are shown for different time instants. In the first snapshot ($t = t_0$) strong vorticity is generated by the breaking on the right side of the tank. This same vorticity advected along the free surface restrains the formation of a left side breaking ($t = t_0 + T/2$) and later of the right side one ($t = t_0 + T$). Once it has been dissipated there is a new left fragmentation of the free-surface at $t = t_0 + 3T/2$. This implies that the vorticity has become the modulating factor of the sloshing flow that does not damped out in time.

Conclusions and perspectives. Even though the present formulation of the SPH method still presents some lacks in the modeling of surface tension, viscosity and turbulence, it has proved its ability to capture the physical features of violent fluid motions both around and on a vessel. Besides the extension to the full Navier-Stokes formulation another straightforward step we are going to take is the extension of the numerical solver to the modeling of 3D problems. These will allow a further and more detailed analysis of other marine hydrodynamic problems.

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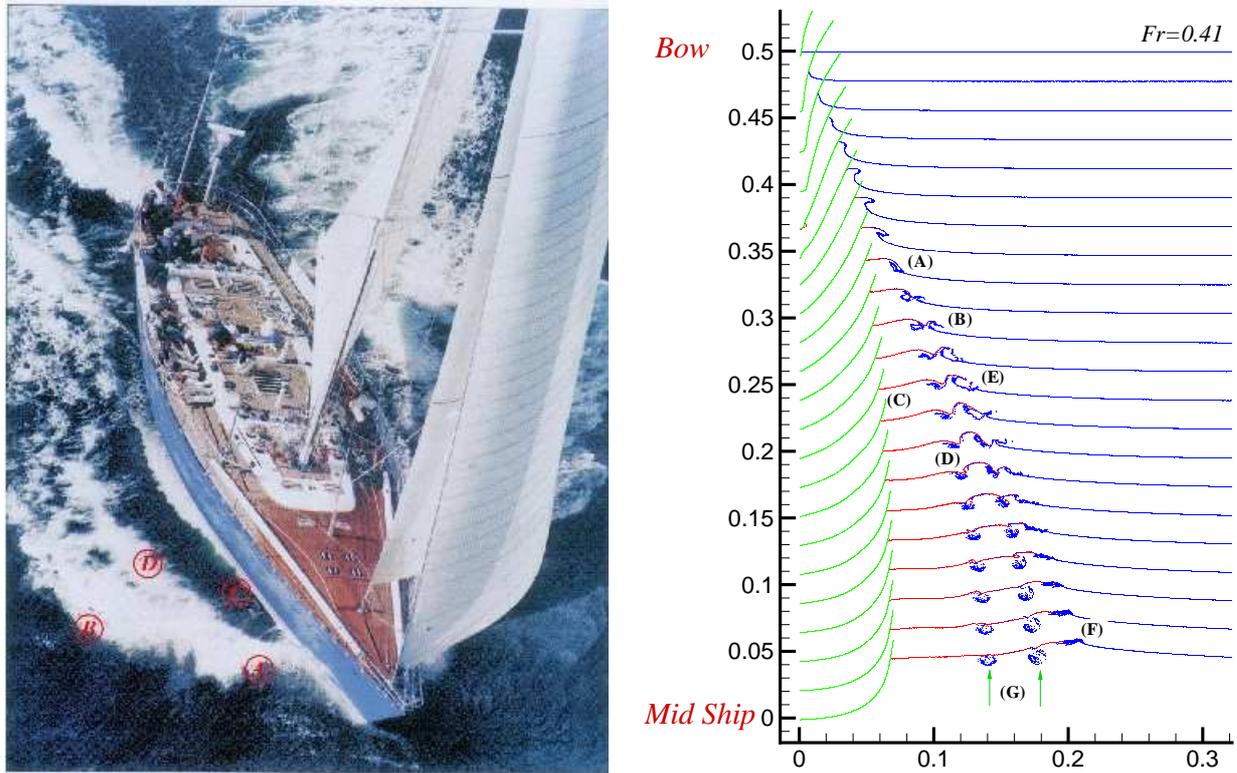


Figure 0.1: Left: Bow-breaking waves for a yacht during *Bermuda race* competition (1985). Right: Three-dimensional reconstruction of the wave pattern around a frigate advancing with $Fr=0.41$. A: plunge point of the jet emerging from the bow splash and starting of the first breaking cycle. B: splash-up. C: plateau between the breaking front and the hull. D: scar left on the free surface. E: second breaking cycle. F: quasi-steady spilling breaker. G: tracks of longitudinal vortical structures from first and second breaking cycles.

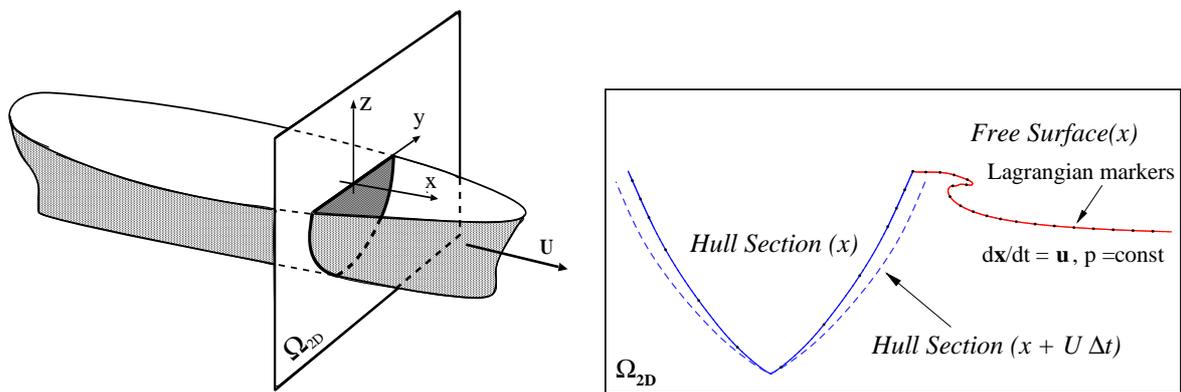


Figure 0.2: Qualitative sketch of the $2D+t$ approximation for the steady three-dimensional flow around a ship with constant forward speed \vec{U} . Left: 3D ship problem. Right: equivalent unsteady 2D problem ($2D+t$).

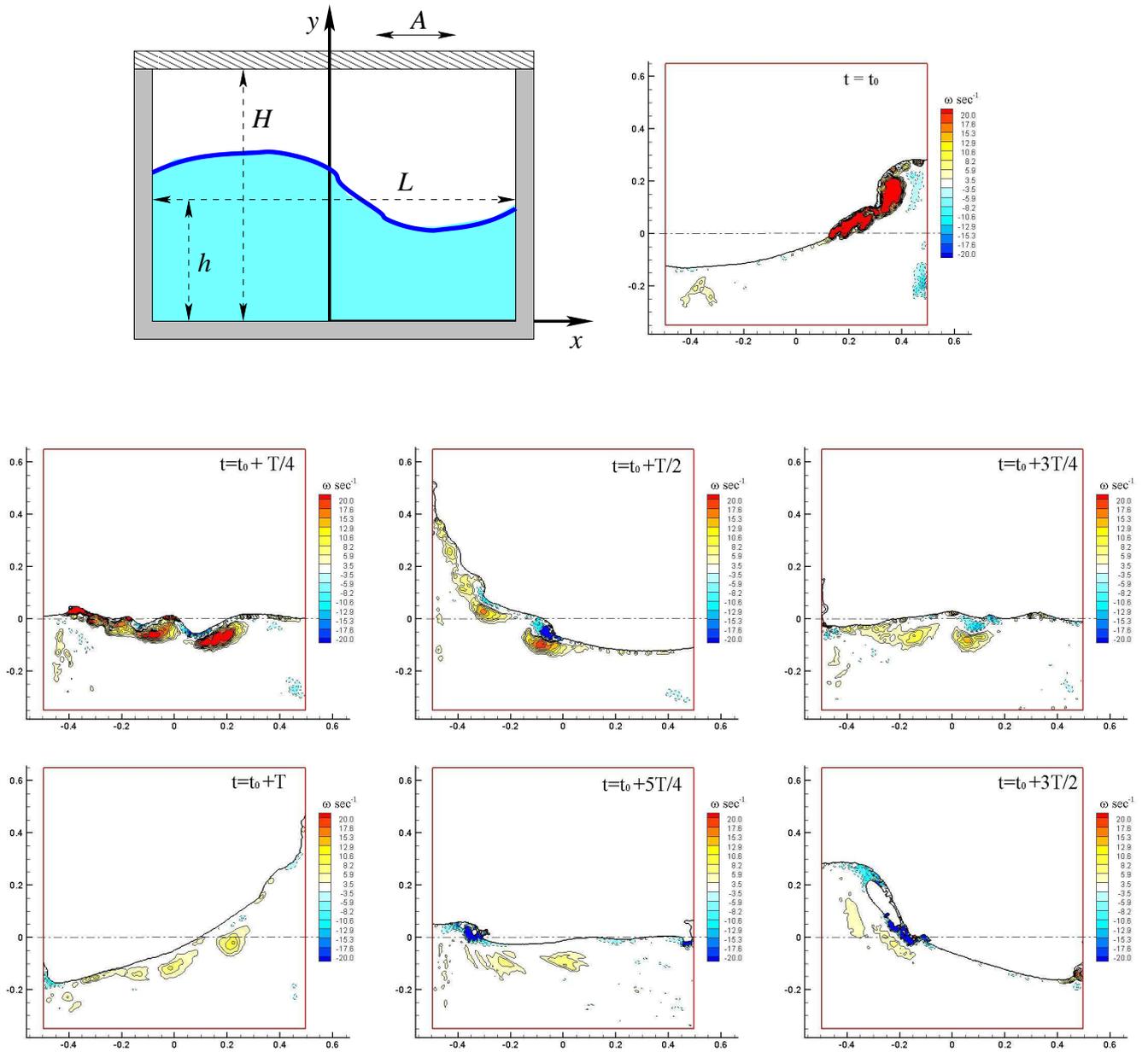


Figure 0.3: Top-left plot: Sketch of the sloshing problem and adopted nomenclature. From top to bottom and from left to right: Evolution of the filled water during a sloshing phenomenon: case with sinusoidal motion with excitation amplitude and period respectively $A = 0.05L$, $T = 0.945T_1$ and filling height $h = 0.35L$. Free-Surface configurations and contour levels of the vorticity calculated through SPH method.