Numerical methods for SAR evaluation in biological bodies

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Abstract.

This paper deals with the development of numerical schemes devoted to the study of the energy absorption from biological body exposed to an external source of electromagnetic (EM) field. The specific absorption rate (SAR) distribution in a human head is evaluated by employing two different numerical schemes: the Method of Moments and the Finite Difference Time Domain method. The first one solves the EM problem described by Greens integral equations in frequency domain by replacing human tissues with equivalent free-space current density, while for the second one a direct numerical discretization of Maxwell’s equations and constitutive relations in time domain are used.

Keywords: SAR evaluation, FDTD method, Method of Moments.

1. Introduction

In the last years, one of the most discussed topics deals with the exposure of human body to electromagnetic fields and their possible effects on the health. During the last century, the exposure to electromagnetic field, in the range of frequencies from zero to some hundreds of GHz, has extremely increased, along with the scientific and technological progress. The remarkable interest caused by the problem related to the social environment, is balanced by equal attention related to the exposures in working places, for which the scientific problem has been set up and firstly warned by careful standards.

2. BIOLOGICAL EFFECTS OF EM FIELDS

The interactions between EM radiation and biological tissues are not expressible only by an elementary process: the interactions and their effects on biological matter are different, depending on many variables and parameters. It is useful to divide these interactions in three categories:
• At the first level, we can consider *perturbations* on biological structures (little or big molecular structures) without perceptible or clinic effect;

• *Reversible biological effect:* at this level, a modification in the biological structure is evident with or without instrumental application. The effect ceases immediately or shortly after the extinction of the stimulus;

• *Irreversible biological effect:* at this level, biological damage, clinically evident, persists for long times even after the extinction of the stimulus. Morphological and/or functional changes are evident when the biological effect cannot be compensated by biological defences or adaptation systems.

About the effects of EM field on the biological matter, a distinction based on the chemical and physical interactions can be made: At lower frequencies (< 1MHz), the prevalent effect is the induction of electric currents in some tissues (nerves, muscles); a such interaction causes the so-called non thermal effects. At frequencies over 1MHz, the EM field gives energy to tissues by the rapid oscillation of ions and water molecules, originating heat and thermal increase. Above 10 MHz, the thermal effect caused by RF and microwaves is the only predominant effect. The effects of this heating in human tissues are cataract, scald, electroshock, chromosome alterations. The absorption at frequencies greater than 10 MHz regards only the epidermic layer.

### 3. PARAMETERS RELATED TO THERMAL BIOLOGICAL EFFECTS

With respect to thermal effects due to EM field exposition, the *Specific Absorption Rate* (SAR) is referenced as the more significant quantity to be evaluated. It is expressed as the ratio between the power absorbed and the radiated tissue mass. From literature, it has been proved that SAR depends on these factors:

1. Dielectric properties of the radiated object;
2. Object size with respect to radiation wavelength;
3. Electric features of the environment where the exposition to the EM field takes place;
4. EM field time variation or modulation;
5. EM field direction and polarization.

Energy absorption by biological tissues is a function of the ratio between the radiated body length (D) and the EM field wavelength (\(\lambda\)); the maximum absorption takes place when the incident radiation has the wavelength near to the body dimension. In the specific:

- If \(D \ll \lambda\), the energy absorption is low; the energy distribution can be considered as constant for the exposed volume.
- If \(D \approx \lambda\), the energy absorption is greater. The energy distribution is not uniform, but is increased on the surface of radiated bodies, thus creating hot spots.
- If \(D \gg \lambda\), the absorption is selective and limited to the exposed zone surface. This is the case of the authors study.

Therefore, SAR is depending on the EM field frequency: the human body has an inhomogeneous absorption, the maximum values being located in the range 30-300 MHz; for an average-sized man, the maximum absorption is located in the range 70-100 MHz\(^1\). By the actual knowledge in medical field, expositions implying a SAR < 4W/kg are considered as not dangerous.
4. NUMERICAL SCHEMES

In order to simulate complex EM environments, different numerical schemes can be usefully employed, in time or in frequency domain. Besides, the use of efficient numerical tools for the prediction of EMI, enables to avoid expensive experimental facilities. The key-point is the right choice of the modeling method for the correct reproduction of the biological body. In this paper, both the Method of Moments (MoM) and the Finite Difference Time Method (FDTD) are implemented, in order to evaluate the SAR for a human head hit by an EM wave. The results are shown and compared in the last section.

4.1. METHOD OF MOMENTS

The first employed numerical model is based on the MoM\(^4\). Consider a biological body of an arbitrary shape, with constitutive parameters \(\epsilon, \sigma\) and \(\mu\), illuminated by an incident EM wave. The induced current gives rise to a scattered electric field, which may be accounted for by replacing the body with an equivalent free-space current density, given by:

\[
\vec{J}_{eq}(\vec{r}) = [\sigma(\vec{r}) + j\omega(\epsilon(\vec{r}) - \epsilon_0)] \left[\vec{E}(\vec{r}) \right]
\]

With this equivalent current density, and the values of the incident field, the knowledge of the electric total field \(\vec{E}\) is obtained by solving the Volume Type Electric Field Integral Equation\(^3\) in frequency domain:

\[
\vec{E}(\vec{r}) = \vec{E}^i(\vec{r}) - j\omega\mu \int_V \tilde{G}_e(\vec{r}, \vec{r}') \vec{J}_{eq}(\vec{r}') d\vec{v}'
\]

where

\[
\tilde{G}_e(\vec{r}, \vec{r}') = \left(\hat{I} + \frac{1}{k^2} \nabla \nabla\right) \tilde{g}(\vec{r}, \vec{r}')
\]

\(\hat{I}\) is the dyadic identity and

\[
\tilde{g}(\vec{r}, \vec{r}') = \frac{e^{-jk|\vec{r} - \vec{r}'|}}{4\pi |\vec{r} - \vec{r}'|}
\]

is the Green scalar function.

The reader is invited to refer to work of Wang\(^4\) for a more detailed description of the adopted numerical algorithm. The first experimental set-up of the system under study is sketched in Fig. 1. A human head model is realized, by using 1718 cells. The relative electric constant and dielectric conductivity of the cells depend on the frequency of the incident wave and on the biological material, as shown in Fig. 2. At the frequency of 900 MHz the values are reported in Table 1.

Once the values of the electric field are known for each cell, SAR is evaluated by the formula

\[
SAR = \frac{\sigma |\vec{E}|^2}{\rho}
\]

and the results are shown in Fig. 3 for the median vertical layer of the head hit by a 1V/m plane wave.

In Fig. 4 the results for an oscillating dipole hitting the human head are shown. The dipole is simulated by a 0.11 A sinusoidal current (corresponding to a radiated power of 0.6 W) and is 3.45 cm distant from the side of the head. The frequency is 1800 MHz.
4.2. **FINITE DIFFERENCE TIME DOMAIN METHOD**

In order to evaluate the electric field time propagation and the instantaneous SAR distribution, the FDTD method has been used. This scheme employs the Yee basic lattice to describe the problem domain in which the electric and the magnetic field are computed\(^5\); this feature allows to easily model complex 3D structures, but as consequence the whole spatial region has to be simulated, thus requiring significant computational resources. It solves, via discrete approximation, the Maxwell’s curl equations:

\[
\nabla \times \vec{E}(\vec{r}, t) = -\mu(\vec{r}, t) \frac{\partial \vec{H}(\vec{r}, t)}{\partial t}
\]

---

**Table 1.** Electric features of biological tissues at the frequency of 900 MHz.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>(\epsilon_r)</th>
<th>(\sigma)</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([\text{S/m}])</td>
<td>([\text{kg/m}^3])</td>
<td></td>
</tr>
<tr>
<td>Muscle</td>
<td>53.5</td>
<td>1.38</td>
<td>1040</td>
</tr>
<tr>
<td>Fat</td>
<td>6.2</td>
<td>0.11</td>
<td>920</td>
</tr>
<tr>
<td>Grey matter</td>
<td>44.1</td>
<td>0.67</td>
<td>1040</td>
</tr>
<tr>
<td>Eye</td>
<td>63.3</td>
<td>1.32</td>
<td>1010</td>
</tr>
<tr>
<td>Bone and cartilage</td>
<td>7.3</td>
<td>0.10</td>
<td>1810</td>
</tr>
<tr>
<td>White matter</td>
<td>39.4</td>
<td>0.48</td>
<td>1040</td>
</tr>
</tbody>
</table>
Fig. 3. SAR distribution in the central vertical layer of the head illuminated by a 1 V/m plane wave at the frequency of 900 MHz.

Fig. 4. SAR distribution in the horizontal layer marked in Fig. 1. The head is illuminated by an electric dipole with 0.11 A current and 3.3 cm in length.

\[ \mathbf{\nabla} \times \mathbf{H}(\mathbf{r}, t) = \epsilon(\mathbf{r}, t) \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} + \sigma(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \]

In a Cartesian coordinates system, neglecting the second term on the right hand of Eq. 5 which accounts for the current, the previous vector equations can be split into six scalar ones:

\[ \frac{\partial H_x}{\partial t} = -\frac{1}{\mu_0} \left[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right] \]

\[ \frac{\partial H_y}{\partial t} = \frac{1}{\mu_0} \left[ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right] \]

\[ \frac{\partial H_z}{\partial t} = -\frac{1}{\mu_0} \left[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right] \]

\[ \frac{\partial E_x}{\partial t} = +\frac{1}{\epsilon_0} \left[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] \]

\[ \frac{\partial E_y}{\partial t} = +\frac{1}{\epsilon_0} \left[ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right] \]

\[ \frac{\partial E_z}{\partial t} = +\frac{1}{\epsilon_0} \left[ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right] \]
Successively these equations can be handled by a finite difference scheme, in which the differential operators are approximated with difference between two near homonymous fields, either in space or in time (for easiness’s sake, only the reformulation of Eq. 6 is reported):

\[
\frac{H^{n+1}_{x} (i, j + 1/2, k + 1/2) - H^{n}_{x} (i, j + 1/2, k + 1/2)}{\Delta t} =
\]

\[
= -\frac{1}{\mu_0} \left[ \frac{E^{n+1/2}_{z} (i, j + 1, k + 1) - E^{n}_{z} (i, j + 1/2, k + 1)}{\Delta y} \right] +
\]

\[
+ \frac{1}{\mu_0} \left[ \frac{E^{n+1/2}_{y} (i, j + 1, k + 1/2) - E^{n}_{y} (i, j, k + 1/2)}{\Delta z} \right]
\]

By using this method the EM problem is led to a system of equations, which is solved step by step via an explicit algorithm under the prescribed boundary and initial conditions. Time step and space step are bound each other by the Courant-Friedrichs-Levy condition:

\[
\Delta t \leq \frac{\Delta x}{\sqrt{n}c_0}
\]

in which \( n \) is the dimension number of the problem.

The simulation of transient radiated EM field constitutes an open boundary problem, so the perfectly matched layer (PML) is used in order to enforce the open region constrains and to limit the extent of the solution region without requiring cumbersome resources. The same model of the human head has been used for this simulation. For the layer marked in Fig. 5, the SAR values at various time steps are shown in Fig. 6 and Fig. 7. Both spatial distributions are similar, but they are to be evaluated at different scales depending on the different instantaneous values of the source current.

![Human head model. The SAR levels in the marked layer are shown in Fig. 4, in Fig. 6 and Fig. 7.](image)

Conclusions

In this paper the effectiveness of two different numerical methods for evaluation of SAR in a human head has been presented. As example, by using the MoM, the values of SAR
Fig. 6. Instantaneous SAR distribution in the horizontal layer marked in Fig. 5. The head is illuminated by an electric dipole with 0.11 A current and 3.3 cm in length. t=3.69 ns.

Fig. 7. Instantaneous SAR distribution in the central horizontal layer of the head is illuminated by an electric dipole with 0.11 A current and 3.3 cm in length. t=9.17 ns.

for an incident plane wave at the frequency of 900 MHz and for an electric sinusoidal dipole at 1800 MHz have been evaluated. The FDTD method has been used to show the instantaneous distribution of SAR while the dipole wave hits the head. Both schemes allow the direct evaluation of the electric field in different layers of the head, showing the cells which are subject to the greatest values of electric field as the EM wave propagates. The FDTD method seems to be more useful in presence of a non periodic source, because of the huge time requested by the frequency analysis and the subsequent Fourier anti-transformation when using the MoM.

REFERENCES
1. Guidelines for Exposure To Time-Varying Electric, Magnetic and Electromagnetic Fields (up to 300 GHz); ICNIRP (Health Physics 74: 494-522, 1998)