

## Quantifying Dissipation

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Reversible thermodynamic processes are convenient abstractions of real processes, which are always irreversible. Approaching the reversible regime means to become more and more quasistatic, letting behind processes which achieve any kind of finite transformation rate for the quantities studied. On the other hand studying processes with finite transformation rates means to deal with irreversibilities and in many cases these irreversibilities must be included in a realistic description of such processes. There are various approaches how to not neglect finite times and rates while not being slain by the real worlds complexity. Endoreversible thermodynamics is a non-equilibrium approach in this direction by viewing a system as a network of internally reversible (endoreversible) subsystems exchanging energy in an irreversible fashion.

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### 1. Introduction

Real energy conversion processes are irreversible, and thus unavoidable losses occur as entropy is produced. These losses limit the efficiency of such processes and new bounds on the performance of heat engines appear. Thus equilibrium thermodynamics which compares real processes to reversible processes proceeding without losses at an infinite slow speed leads to a valid upper bound for efficiency. However they can not provide a sensible least upper bound for real irreversible processes and thus they may not be good enough to be a useful guide in the improvement of real processes. An example is the often used Carnot [1] efficiency:

$$\eta_C = 1 - T_L/T_H. \quad (1)$$

It gives the fraction of the heat which at most can be converted to work in any engine using heat from a hot reservoir at temperature  $T_H$  and rejecting

some of the heat to a reservoir at lower temperature  $T_L$ . Real heat engines, for example, seldom attain more than a fraction of the reversible Carnot efficiency.

Engineers tried to close this discrepancy between real process and limiting reversible process by improving their design, specific to certain devices or processes. But despite all technological progress in engineering, the gap remains, and it has to remain due to the irreversible nature of real processes.

Thus the principle questions are “What are realistic bounds for thermodynamic processes preformed in finite time?” and “What are valid process paths to achieve this optimal process?”

Already about fifty years ago the effect of finite heat transfer rates came into the focus of efficiency considerations for heat engines [2–6]. They investigated the effect of finite heat transfer on the power output of an otherwise reversible power plant. They discovered that the efficiency at the maximum power point,

$$\eta = 1 - \sqrt{T_L/T_H}, \quad (2)$$

is considerably lower than the corresponding Carnot efficiency. However, at that time this research activities did not arrest much attention.

After Curzon and Ahlborn 1975 re-discovered [7] the efficiency expression (2), which they found in remarkable agreement with the performance data of real power plants, the framework of finite time thermodynamics evolved (see for instance [8–11]). One reason was the oil crisis hitting the economy, again forcing the view on efficiency limiting irreversibilities of heat engines. The goal of this framework is to determine performance bounds for thermodynamic processes proceeding in a finite time or with finite rates. It has been applied since then mainly thermal engines or processes. In general the approaches taken were focused on the inclusion of the major loss terms and irreversibilities. In that way the models were kept simple while at the same time the results stayed realistic enough to provide useful insights. One particularly effective development is the use of endoreversible models [12–15]. For instance Curzon and Ahlborn [7] used a very simple endoreversible model to estimate efficiencies for power stations and compared those to measured data. The agreement was quite good compared to the values obtained from the usual Carnot-efficiency.

## 2. Endoreversibility and endoreversible systems

An endoreversible system consists of a number of subsystems which interact with each other and with their surroundings. We choose the *subsystems*

so as to insure that each one undergoes only reversible processes. All the dissipation or irreversibility occurs in the *interactions* between the subsystems or the surroundings. An endoreversible system is thus defined by the properties of its subsystems and of its interactions. We call processes of such systems endoreversible process.

If a subsystem is for instance a spatially uniform working fluid, than the requirement that it undergoes only reversible processes means that it is always in internal thermodynamic equilibrium. But subsystems can be also more aggregate objects, namely engines (or more general energy transformation devices). If for instance such an engine takes in heat at temperature  $T_H$  and converts it into work and heat discharged at temperature  $T_L$ , then endoreversibility requires its efficiency  $\eta$  to be the Carnot efficiency  $\eta_C = 1 - T_L/T_H$ . This will become more apparent in our first example.

A complete definition and formal description of endoreversibility and endoreversible systems can be found in the previously published review articles [15] and [16], where also an overview on endoreversible techniques and systems given.

### 2.1. An introductory example

As a simple introductory example we consider the Novikov engine [2, 3], a simplified version of the Curzon-Ahlborn engine treated later. The Novikov engine is a continuously operating, reversible Carnot engine with the internal temperatures  $T_{iH}$  and  $T_{iL}$ . It is in direct contact with the external low temperature heat bath at  $T_L$  and is coupled to an external high temperature heat bath at  $T_H$  through a finite heat conductance  $K$  (see Figure 1). The heat baths are both considered to be infinite such that the in- and outflux of energy does not change their temperatures.

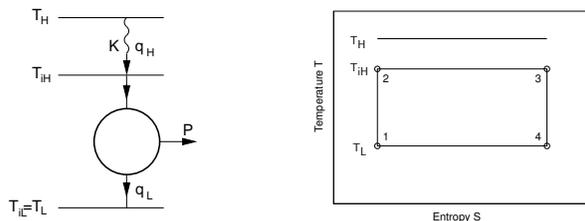


Fig. 1. Model of the endoreversible Novikov engine with finite heat conduction  $K$  to the high temperature heat reservoir (*left*). TS-diagram of a Carnot cycle with a temperature difference to the high temperature heat reservoir (*right*).

The question is now how the irreversibility introduced by the finite heat conductance influences the performance of the engine. Does it for instance have an effect on the efficiency of the system?

We first note that the total heat flux through the system is limited, and thus the power produced by the engine is limited as well. To characterize the performance of this endoreversible system in more detail we want to determine the maximum power available and the efficiency at the operating point of maximum power.

Due to the finite heat conductance heat is only transported to the Carnot engine, if its high temperature  $T_{iH}$  is lower than the bath temperature  $T_H$ . The heat flux  $q_H$  transported from the heat bath to the engine is assumed to be proportional to the temperature difference (Newtonian heat conduction)

$$q_H = K(T_H - T_{iH}). \quad (3)$$

At the low temperature side the heat can be discharged to the heat bath at  $T_L$  without any resistance. Thus this interaction between heat bath and engine is characterized not by a transfer law but by the requirement that the lower temperature  $T_{iL}$  of the Carnot engine is the same as the bath temperature

$$T_L = T_{iL}. \quad (4)$$

We note that the Carnot engine is characterized by three energy fluxes and two temperatures:  $q_H$  enters the engine at temperature  $T_{iH}$ , the heat flux to the low temperature heat bath  $q_L$  leaves the engine at temperature  $T_{iL}$ , and  $P$  is the power delivered by the engine. The heat baths are described by  $(T_H, q_H)$  and  $(T_L, q_L)$  respectively, and the heat conduction contains the parameter  $K$ . All the variables (energy fluxes, temperatures, and  $K$ ) are related by the interaction between the heat bath and the engine and by the constraints coming from the endoreversibility of the Carnot engine. As the engine operates continuously in a steady state, all the energy fluxes have to balance

$$0 = q_H - q_L - P. \quad (5)$$

In addition, as the engine operates reversibly the entropy fluxes to and from the engine have to cancel

$$0 = \frac{q_H}{T_{iH}} - \frac{q_L}{T_{iL}}. \quad (6)$$

Solving now for  $P$  we obtain

$$P = q_H \left( 1 - \frac{T_{iL}}{T_{iH}} \right) = K(T_H - T_{iH}) \left( 1 - \frac{T_L}{T_{iH}} \right). \quad (7)$$

For given temperatures of the heat baths and given  $K$ , the flow of heat through the engine and the power produced by the inner Carnot engine will depend only on the operating temperatures of the Carnot engine. As  $T_H$  and  $T_L$  are fixed, the only control to influence the overall performance of the endoreversible engine is  $T_{iH}$ , and we find the power  $P$  as a function of  $T_{iH}$  only. Equation (7) alone is thus characterizing the entire endoreversible Novikov engine with Newtonian heat conduction.

The maximum power is determined by differentiation with respect to  $T_{iH}$

$$0 = \frac{dP}{dT_{iH}} = K \left( \frac{T_H T_L}{T_{iH}^2} - 1 \right), \quad (8)$$

from which we find  $T_{iH} = \sqrt{T_H T_L}$ . Operating with this temperature the maximum power is

$$P_{max} = K \left( \sqrt{T_H} - \sqrt{T_L} \right)^2 \quad (9)$$

and the efficiency in terms of the bath temperatures is

$$\eta(P_{max}) = 1 - \frac{T_{iL}}{T_{iH}} = 1 - \sqrt{\frac{T_L}{T_H}}. \quad (10)$$

The reader should note the remarkable fact that this efficiency does not depend on the size of the heat conductance  $K$ . Also note that this efficiency is not a bound for heat engines operating not at the maximum power point.

This simple example has shown how with a relatively modest effort new and interesting results can be obtained for the performance of heat engines operating out of equilibrium.

### 3. Optimal Paths for Internal Combustion Engines

The application of finite-time thermodynamics to combustion engines is of course not restricted to the analysis of the efficiencies. As important is the question what the optimal thermodynamic paths of such engines are. The path optimization shows which loss term can be most easily reduced, and how close real engines approach these performance bounds. Dynamic endoreversible models of internal combustion engines, especially Diesel and Otto engines, have been first investigated by Mozurkevich and Berry [17],

by Hoffmann, Watowich and Berry [18], and later extended by Blaudeck and Burzler [19, 20].

Mozurkevich and Berry [17] investigated an internal combustion engine based on an Otto cycle. Their objective was to find the optimal piston path for which the output of work is maximized for a fixed cycle time and a given amount of fuel. The model is based on a four stroke Otto cycle with several sources of irreversibility such as piston friction and heat leak. The models employed to describe the losses were chosen such that they adequately model the qualitative behavior and the total magnitude of these losses.

Losses due to friction are approximated by a friction force linear in the piston velocity. For heat conduction coefficient  $\kappa$  and cylinder diameter  $b$  the rate of heat leak  $q(x, t)$  at piston position  $x$  is given by:

$$q(x, t) = \kappa\pi b(0.5b + x)(T - T_{\text{ex}}). \quad (11)$$

To find the optimal cycle for a given total cycle time one has to calculate the optimal path for each stroke as a function of time needed for this stroke.

Mozurkevich and Berry obtained a number of highly interesting results. The increase of effectiveness is calculated for several sets of parameters and different values for the constraints of piston acceleration. It turns out that compared to a conventionally operated piston the magnitude of achievable increase of effectiveness due to path optimization is about 10%. Especially for those parameter sets with large heat leaks the optimal movement requires a fast expansion to avoid heat losses during the expansion. For details see ref. [17].

The main difference of the Diesel combustion engine to the Otto cycle is due to the finite combustion rate of the fuel which progresses also during the power stroke. This leads to a higher in principle efficiency of the Diesel engine. So it is very interesting to analyze a Diesel engine model to determine performance limits for this type of process. Hoffmann, Watowich, and Berry [18] performed such an analysis. The optimal piston motion showed a very surprising result: The piston should not move at all for the first part of the power stroke. This behavior seems highly wasteful, as it increases the frictional losses due to the higher velocity needed in the remaining time for the power stroke. However it turned out that due to the piston remaining fixed the temperature of the working fluid can increase higher this way. This in turn means that the available heat energy of the fuel is provided to the system with higher exergy or availability content. This shows eventually up in a higher work and power output. Again the engine efficiency was about 10% higher than for a conventionally operated piston. Later Burzler,

Blaudeck and Hoffmann [20] moved the model even closer to real engines by investigating different heat loss models.

As an alternative to control theory a Monte-Carlo method can be used [19] as effectively. The results for the optimal operation of the compression and power stroke of a Diesel engine are shown in Fig. 2, where both strokes were optimized together using the Monte-Carlo method. Again the optimal power stroke begins with a short delay, where the piston remains at its extremal position. So this interesting behavior, which increases losses due to heat leak and friction but increases the temperature of the working fluid and the maximum availability of the system, remains unchanged.

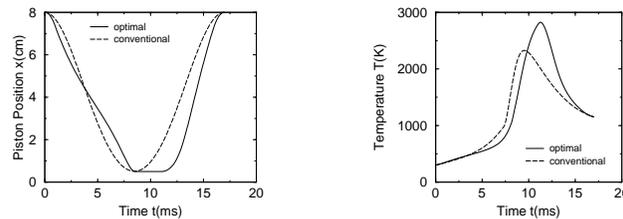


Fig. 2. Optimal and conventional piston path of the compression and power strokes in a Diesel engine. Note the initial stand-still of the piston during the power stroke (*left*). Temperature of the working fluid for optimal and conventional path (*right*).

#### 4. How realistic are endoreversible model systems?

An interesting question is whether the comparatively simple models presented above can be a good approximation for complex real systems. We thus examined whether a full featured engineering-style model can be modelled by an endoreversible model containing only a few components.

The simple models were originally introduced to provide a qualitative insight, so it was an open question to what extent such models can also be used for a more quantitative description. Is for instance the structure of a Curzon-Ahlborn engine, which includes only dissipative losses due to finite Newtonian heat conductances, rich enough to model also heat losses of a much complexer nature by an appropriate parameter choice? Or is there a way to describe a heat engine with its performance dependence on the operating speed by a model without an explicit engine cycle? Answering such questions will provide the basis on which the range of the usefulness of simple thermodynamic models can be judged more precisely.

To that end we studied a comparison of a benchmark internal combus-

tion engine based on an Otto cycle to its much simplified endoreversible counterpart [21, 22]. The performance features of an elaborate engine simulation were studied including a large number of important dissipative losses and it was tried to establish a relation to the parameters of a endoreversible model engine. The particular goal was to determine in what way these parameters depend on the details of the benchmark engine.

The counterpart to the benchmark engine, the simple endoreversible model, consisted of a Carnot-engine with heat transfer and heat leak as proposed in [23, 24]. It is basically a Novikov-engine like the one in section 2.1 with an additional heat leak.

The work characteristics of the two engines have been compared, where the energy and entropy balance as measures of comparison was used. We found that the simple model captures the important features of the benchmark engine well and with some surprisingly simple relations between the model parameter and the engine data.

The benchmark engine is a 1-cylinder 4-stroke Otto engine set up as schematically shown in fig. 3. The chosen parameters reflect typical values for Otto engines. All major functional parts and processes of the engine are taken into account, namely mechanics, gas dynamics within cylinder, gas exchange, combustion, heat contact, and load. For a fully detailed description of the process models and parameters used see [21] or [22].

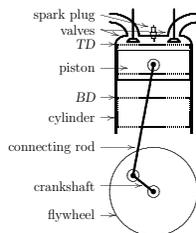


Fig. 3. Setup of the combustion engine (schematic)

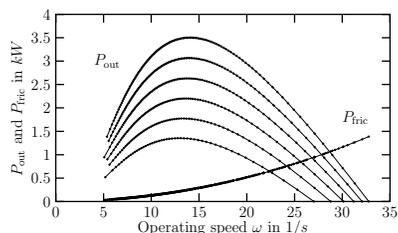


Fig. 4. Produced power and friction loss of the benchmark engine for various inlet pressures. The group parameter of these curves is the inlet pressure and the variation of the operating speed at constant inlet pressure is a result of the variation of the load.

Figure 4 shows the produced power as a function of the operating speed  $\omega$ . Note the smooth dependence of the power on the decreasing load. The power maximum is for lower inlet pressure at smaller  $\omega$ . In addition the

power  $P_{\text{fric}}$  due to frictional losses is shown. Figure 5 shows the entropy flows as a function of the operating speed  $\omega$ . Again the curves are smoothly dependent on changes in inlet pressure and load. The entropy flows out are considerable larger in size than the flows in, indicating the amount of entropy produced.

Finally figure 6 shows the produced power as a function of the efficiency. Note that maximum power is achieved over the whole operating range at higher operating speed than maximum efficiency. Such a behavior has already previously been observed for heat engines [24].

The open question is now whether this dependence between produced power and efficiency can quantitatively be reproduced by a simple endoreversible engine model with an appropriate parameter choice.

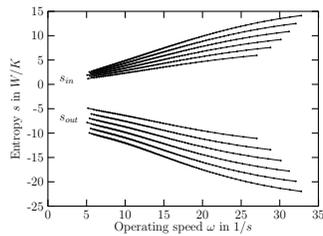


Fig. 5. The entropy flows in the benchmark engine for various inlet pressures (group parameter).

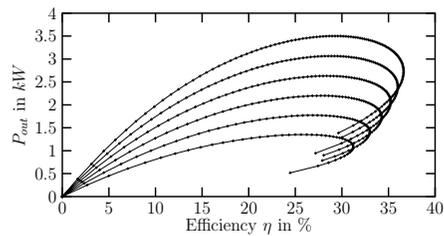


Fig. 6. Produced power vs. efficiency within working range. Again, the inlet pressure is the group parameter. Note that with increasing operating speed the curves show first a maximum in efficiency and then in power.

Our definition of equivalence between the model and the benchmark engine requires that the energy as well as the entropy flows match. With this constraint, the bath temperatures for the endoreversible model can be determined and are shown in figure 7. As the benchmark engine is controlled by load and inlet pressure the graph shows the temperatures for various combinations of load and inlet pressure as a function of the operating speed corresponding to the respective load/pressure combination. The surprising result is that within reasonable accuracy the temperatures are not explicitly dependent in the control parameters load and inlet pressure, but are dependent on a particular combination of these. One can easily see that the temperature values are mainly dependent on only one parameter which is the operating speed.

A comparison of the original and reproduced working behavior is shown

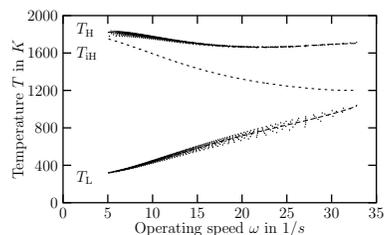


Fig. 7. Reservoir temperatures: benchmark data and model fits. Note that  $T_H$  is approximately constant while  $T_L$  is linearly increasing.

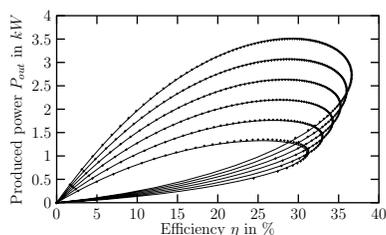


Fig. 8. In this produced power vs efficiency plot the original benchmark engine data (points) and the reproduction (lines) by the Novikov engine with heat leak are compared. Note how well the much simpler endoreversible model reproduces the complicated Otto engine simulation.

in figure 8; there the points represent the original data from the benchmark engine and the lines represent the reproduction by the endoreversible model. Because of the desired features of the approximation, small differences between the benchmark and model engine's data are unavoidable. Although these differences are visible within figure 8 the general accuracy achieved can be considered quite high.

The overall quality with which the work characteristics of the benchmark engine can be reproduced is very high. Thus for this case study the question whether the structure of the Novikov engine with heat leak is rich enough to also quantitatively model much more sophisticated engine simulations can be answered affirmative under the condition that the frictional losses are part of the overall power produced.

## 5. Power-Control of internal combustion engines

The above discussed modelling can be successfully used to investigate effects of modified operating scenarios for real engines.

One potentially energy conserving idea was to develop a new method controlling an engine's power. Of course the power is controlled by the amount of gas load in the cylinder. Up to now that is mainly controlled by adjusting the throttle valve (I). Recently, the BMW company proposed the method of varying the time at which the inlet valve closes instead of throttling the gas (II). A third proposed method is to always fill the cylinder with the maximum amount of air and afterwards opening the exhaust valve for removing the superfluous amount of it (III). Of course this method is only applicable in conjunction with diesel or gasoline direct injection engines

for obvious reasons.

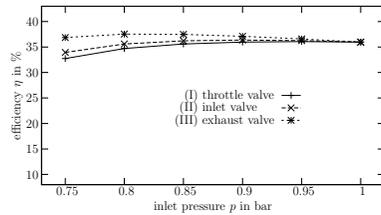


Fig. 9. The engine's efficiency over inlet pressure at constant load for the different methods of power-control. Note the considerable efficiency gain potential for method (II) and especially (III) in the important area of reduced power.

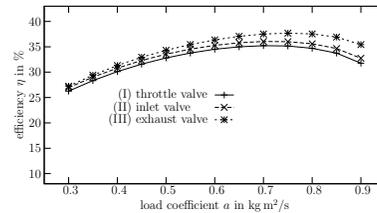


Fig. 10. The engine's efficiency over load at constant (medium) inlet pressure for the different methods of power-control. Note the considerable efficiency gain potential for method (II) and even more (III) especially at higher load.

The question we analyzed was how these methods perform regarding efficiency, especially in the important region of partial load. The results of our work are shown in figures 9 and 10 while varying load and accelerator position. Only variant (I) really has a variable inlet pressure, in the other cases the valve timings are chosen to match the first ones speed and power output. We found out that at maximum up to 4% points could be saved by (III) compared to (I).

The next question arising is certainly what particular effect the efficiency gain is based on. Further work on this topic will be published in future.

## 6. Conclusion

We considered endoreversible systems ranging from general thermodynamic cycles with different model assumptions to semi-realistic combustion engine models. Various examples showed how different techniques are applied to determine performance optima and the corresponding optimal process path for such systems.

Endoreversible thermodynamics in our view is the successful attempt to include irreversibilities and dissipative processes into the description of thermodynamic processes, while at the same time preserving the advantages of classical reversible thermodynamics. The central idea is to think of a system as a network of subsystems – each undergoing only reversible processes – which exchange energy. All irreversibilities occur only in the interactions between the subsystems. Treating systems in this way one gets one step closer to a realistic description of real dissipative processes.

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