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Approximate solutions to the quantum drift-diffusion model of semiconductors in the presence of external barrier potential

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Abstract.

Approximate solutions to the quantum drift-diffusion model for semiconductors are found by including also an external potential modeling in terior quantum barriers. The system is written as singularly perturbed equations which are solved up to first order in the scaled reduced Planck constant.

Keywords: Approximate solutions, quantum drift-diffusion models, singular perturbations

1. Introduction

Drift-diffusion models[1,2,3] are simple macroscopic descriptions of charge carrier transport in semiconductors widely used in engineering applications even when quantum effects are not negligible. They are given by the balance equation for electron density and/or hole density coupled to the Poisson equation for the electric potential.

A first formulation of the quantum drift diffusion model was based on the Magdelung model of quantum fluids[4,5] . It was derived starting from the Schrödinger equation for a pure state and represents the quantum analogous of the zero-temperature (or pressureless) Euler equations. A more refined derivation has been given in[6] on the basis of thermodynamics considerations, assuming that the electron energy depends also on the gradients of the density. For this reason the model, justified in[7] in the framework of kinetic transport theory, is usually referred to as density gradient models. The only difference with the Magdelung model is a different numerical factor in the quantum potential.

Moreover we recall that more recently, the density gradient model has been also obtained by using the maximum entropy principle[8] in a quantum framework.[9] Several analytical and numerical studies of the resulting quantum drift-diffusion model have been performed (the interested reader is referred to[10,11,12,13] and references therein).

The present paper is focused on the search of exact asymptotic solutions.

In[14] by using a singular

perturbation approach a class of solutions has been found with the aid of sy mmetry group analysis, the

perturbation method being justified because the scaled Planck constant

can be considered as a *small* parameter for typical devices

where quantum effects are relevant.

Here we extend the analysis in[14] including also the internal barri er potential.

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In order to deal with more realistic situations as resonant tunneling diodes whose operating functions are strongly depending on quantum effect, the cas e of a barrier potential of harmonic type will be tackled. It describes the motions of electrons inside a thermal bath of harmonic oscillators.

The paper is organized as follows. In sections 2 and 3 the details of the models and its scaled version are presented. Section 4 is devoted to find solutions by an expansion with respect to the scaled Planck constant.

2. The model

We consider the quantum drift-diffusion model based on the Bohm potential that in the unipolar case reads[10]

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J} = 0, \tag{1}$$

$$\epsilon \, \Delta \Phi = e \, (n - c(\mathbf{x})), \tag{2}$$

where n is the electron density, $\mathbf{J}$ the electron momentum density, ϵ the dielectric constant, e the elementary charge, Φ the electric potential and $c(\mathbf{x})$ the doping concentration that is a function of the position $\mathbf{x}$. ∇ and Δ are the nabla and Laplacian operators, respectively.

The system (1)-(2) is supplemented by a constitutive relat ion for the momentum density $\mathbf{J}$, which is expressed as the sum of a diffusion, a drift an d a quantum term

$$\mathbf{J} = -K \, \nabla(n^{\alpha}) + \mu \, n \, [\nabla(\Phi - V) + \nabla Q].$$

K is the diffusion coefficient, μ the mobility, $V(\mathbf{x})$ is an external potential depending on the p osition $\mathbf{x}$ which models interior quantum barriers, Q is the Bohm q uantum correction given by

$$Q = \frac{\hbar^2}{6 \, m^* \, e} \frac{\Delta \sqrt{n}}{\sqrt{n}}$$

where $\hbar$ is the reduced Planck constant and m^* the effective electron mass. In the original Magdelung model the quantum correction has a factor 2 instead of 6. In commercial simulators it is considered as another fitting parameter, usually assumed to vary from zero to one.

α is a constant satisfying $\alpha \geq 1$. For $\alpha = 1$ we have the isothermal case. For $\alpha > 1$ we have a quantum corrected isentropic model. In commercial simulators α is also considered as another fitting parameter.

As usual we assume that K and μ are related by the Einstein relation

$$K = U_0 \mu,$$

where $U_0 = \dfrac{k_B T}{e}$ is the constant thermal potential with k_B Boltzmann constant and T lattice temperature kept at equilibrium.

The mobility μ is considered to be a function of the modulus $|E|$ of the electric field $E = -\nabla(\Phi - V)$, that is

$$\mu = \mu(|E|).$$

In the one-dimensional case the system (1)-(2) can be rewrit ten in terms of the relevant component of the electric field and
reads

$$n_t + \left[\mu \left(\frac{\hbar^2 n}{6\, m^* e} \left(\frac{(\sqrt{n})_{xx}}{\sqrt{n}} \right)_x - U_0\, \alpha\, n^{\alpha-1} n_x + n\,(\Phi_x - V_x) \right) \right]_x = 0, \tag{3}$$

$$\epsilon\, \Phi_{xx} = e\,(n - c). \tag{4}$$

Equations (3), (4) are considered in the whole space or in a bounded domain and are supplemented by initial and boundary conditions. The general specification of these is itself an open problem. In one dimensional cases, e.g. a n^+-n-n^+ diode, the problem is considered in an interval. At the edges of such an interval, usually ohmic contacts, $n = c$, are assumed for the density and the electric potential is assigned. The latter conditions can be equivalently formulated assigning the electric field at one edge and requiring that the integral of the electric field over the domain representing the device is equal to the difference of applied voltage. Moreover it is customary to require that no quantum effects occur at the contacts and therefore the condition $\Delta\, \sqrt{n} = 0$ is also imposed at the edges.
In our approach we will not impose a priori boundary and initial conditions. Only after the solutions has been obtained, the corresponding boundary and initial conditions will be determined a posteriori and selected the physically relevant cases.

3. Scaling

To justify a perturbation approach for solving the evolution equation we rewrite our system in a
scaled form.

The scaling has the advantage to emphasize the singularly perturbed nature o f the equations.

Our choice for the scaling parameters is as follows :

- the scaling factor for the carrier density is chosen to be the maximum v alue of the doping throughout the device:
  $\bar{n} = \|c(x)\|_\infty$;
- voltages are scaled with respect to the thermal potential: $\bar{\Phi} = \frac{k_B T}{e} = U_0$ which is equal to 0.0259 Volt at room temperature;
- lengths are scaled with respect to the characteristic length L of the device: $\bar{x} = L$;
- mobility is scaled with respect to the low field electron mobility:
  $\bar{\mu} = \mu_0$ (e.g. for silicon about 1400cm^2/V sec).

Accordingly, we define the following adimensional quantities

$$\hat{n} = \frac{n}{\bar{n}}; \quad \hat{\Phi} = \frac{\Phi}{\bar{\Phi}}; \quad \hat{V} = \frac{V}{\bar{\Phi}}; \quad \hat{x} = \frac{x}{\bar{x}}; \quad \hat{\mu} = \frac{\mu}{\bar{\mu}};$$
$$H_0 = \frac{\hbar^2}{6\, e\, m^*\, \bar{x}^2 \bar{\Phi}}; \qquad \lambda^2 = \frac{\bar{\Phi}\, \varepsilon}{e\, \bar{x}^2 \bar{n}}.$$

The scaled equations read

$$n_t + \left[-\mu\, U_0\, \alpha\, n^{\alpha-1} n_x + \mu\, n\, (\Phi_x - V_x) + \mu\; H_0\, n \left(\frac{(\sqrt{n})_{xx}}{\sqrt{n}} \right)_x \right]_x = 0,$$
$$\lambda^2\; \Phi_{xx} = (n - c).$$

In a typical device where quantum effects are relevant, $\bar{n} = 10^{24} m^{-3}$ and $\bar{x} = 10^{-7} m$. In such a case for example in Silicon, one has

$$H_0 = 1.88 \cdot 10^{-4}, \quad \lambda^2 = 1.7 \cdot 10^{-3}.$$

Even in smaller devices with $\bar{x} = 10^{-8} m$, one has

$$H_0 = 1.88 \cdot 10^{-2}, \quad \lambda^2 = 1.7 \cdot 10^{-1}$$

and this justifies a perturbation approach for solving the evolution equations.

4. Solutions via a perturbation approach

We look for solutions of our system in the form $n(t, x, H_0)$, $\Phi(t, x, H_0)$ by supposing n and Φ analytic in H_0. Up to first order in H_0

$$n(t, x, H_0) = n_0(t, x) + H_0\, n_1(t, x) + o(H_0) \tag{1}$$
$$\Phi(t, x, H_0) = \Phi_0(t, x) + H_0\, \Phi_1(t, x) + o(H_0). \tag{2}$$

We substitute formally (1)-(2) in our system and equate to zero the coefficients of zero and first degree in H_0.

Hence $(n_0(t,x),\, \Phi_0(t,x))$ is a solution of our system with $H_0 = 0$, while $n_1(t,x)$ and $\Phi_1(t,x)$ can be obtained from the first order system once n_0 and Φ_0 have been determined.

We analyze the following case, that corresponds to a homogeneous semiconduct or,

$$c = c_0, \quad \mu = \mu_0, \qquad V = v_2 x^2 + v_1 x + v_0$$

where c_0, μ_0, v_0, v_1, v_2 are constants and c_0, μ_0 are positive.

In this case the scaled system becomes

$$\begin{aligned} & n_t + \mu_0 \left(H_0 n \left(\frac{2n n_{xx} - n_x^2}{4n^2} \right)_x - \alpha n^{\alpha-1} U_0 n_x + n(\Phi_x - 2v_2 x - v_1) \right)_x = 0 \\ & \lambda^2\, \Phi_{xx} = n - c_0. \end{aligned}$$

As solution of the classical case ($H_0 = 0$), by requiring that

$$c_0 + 2v_2\lambda^2 > 0, \tag{3}$$

we take

$$n_0(t,x) = c_0 + 2v_2\lambda^2, \quad \Phi_0(t,x) = v_2 x^2 + p_1 x + p_0$$

with p_0, p_1 constants
and look for perturbed solutions of the form

$$\begin{aligned} n(t,x,H_0) &= c_0 + 2v_2\lambda^2 + H_0\, n_1(t,x) + o(H_0), \\ \Phi(t,x,H_0) &= v_2 x^2 + p_1 x + p_0 + H_0\, \Phi_1(t,x) + o(H_0), \end{aligned}$$

where n_1 and Φ_1 must satisfy

$$n_1 = \lambda^2\, \Phi_{1xx}, \tag{4}$$

$$\begin{aligned} & n_{1t} + \mu_0 \left[-\alpha U_0 (2v_2\lambda^2 + c_0)^{\alpha-1} n_{1xx} + (p_1 - v_1) n_{1x} + \right. \\ & \qquad \left. + \Phi_{1xx}(2v_2\lambda^2 + c_0) \right] = 0. \end{aligned} \tag{5}$$

We remark that under the condition (3) $n_0(t,x)$ is positive.

Taking into account (4), the equation (5) becomes

$$\begin{aligned} & \left[\lambda^2 \Phi_{1t} + \mu_0 \left(-U_0 \alpha (2v_2\lambda^2 + c_0)^{\alpha-1} \lambda^2 \Phi_{1xx} + \lambda^2 (p_1 - v_1) \Phi_{1x} + \right. \right. \\ & \qquad \left. \left. + \, (2\lambda^2 v_2 + c_0) \Phi_1 \right) \right]_{xx} = 0. \end{aligned} \tag{6}$$

By seeking solutions of the form $\Phi_1 = u(t,x) + x f(t) + g(t)$ where $f(x)$ and $g(x)$ are functions with no restrictions apart those arising

by initial and boundary conditions, $u(t,x)$ must solve the following equation

$$\lambda^2 u_t + \mu_0 \left(-U_0\alpha(2v_2\lambda^2+c_0)^{\alpha-1}\lambda^2 u_{xx} + \lambda^2(p_1-v_1)u_x + (2\lambda^2 v_2 + c_0)u\right) = 0. \tag{7}$$

Exact solutions to the linear equation (7) can be obtained in sever al ways. In the following, we will show two of them.

Case 1

The equation (7) can be transformed in the heat equation

$$v_\tau = v_{\xi\xi}$$

by using the following transformation[15]

$$t = \frac{\tau}{\alpha\mu_0\, U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}} + K_0, \tag{8}$$

$$x = \xi - \frac{(p_1-v_1)\tau}{\alpha\, U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}} + K_0, \tag{9}$$

$$u = v e^{-\frac{\tau}{\alpha\, U_0\, \lambda^2 (2v_2\lambda^2+c_0)^{\alpha-2}}}. \tag{10}$$

So, if $v(\tau,\xi)$ is a solution of the heat equation, taking into account that

$$\tau = (t-K_0)\alpha\mu_0\, U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}$$
$$\xi = x + (p_1-v_1)(t-K_0)\mu_0 - K_0,$$

a solution of equation (6) is

$$\Phi_1(t,x) = v\left((t-K_0)\alpha\mu_0\, U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}, x + (p_1-v_1)(t-K_0)\mu_0 - K_0\right) e^{-\frac{(2v_2\lambda^2+c_0)(t-K_0)\mu_0}{\lambda^2}} + xf(t) + g(t).$$

For example, if we take the fundamental solution of the heat equation

$$v(\tau,\xi) = Err\left(\frac{\xi}{\sqrt{4\tau}}\right),$$

with Err the error function,
and set $K_0 \le 0$, we obtain

$$\Phi_1(t,x) = Err\left(\frac{x + (p_1-v_1)(t-K_0)\mu_0 - K_0}{\sqrt{4(t-K_0)\alpha\mu_0\, U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}}}\right) e^{-\frac{(2v_2\lambda^2+c_0)(t-K_0)\mu_0}{\lambda^2}} + xf(t) + g(t)$$

and the solution up to the first order in H_0 is

$$
\begin{aligned}
n(t,x,\ H_0) &= (c_0+2v_2\lambda^2) - 4H_0\,\lambda^2\,[x+(p_1-v_1)(t-K_0)\mu_0-K_0]\cdot \\
&\cdot\frac{e^{-\frac{[x+(p_1-v_1)(t-K_0)\mu_0-K_0]^2}{(t-K_0)\alpha\mu_0\,U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}}-\frac{(2v_2\lambda^2+c_0)(t-K_0)\mu_0}{\lambda^2}}}{\sqrt{\pi[\alpha\mu(t-K_0)U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}]^3}}+o(H_0),\\
\Phi(t,x,\ H_0) &= v_2^2x+p_1x+p_0+H_0\,xf(t)+H_0g(t)+\\
&+H_0\,Err\left(\frac{x+(p_1-v_1)(t-K_0)\mu_0-K_0}{\sqrt{4(t-K_0)\alpha\mu_0\,U_0\,(2v_2\lambda^2+c_0)^{\alpha-1}}}\right)e^{-\frac{(2v_2\lambda^2+c_0)(t-K_0)\mu_0}{\lambda^2}}+\\
&+o(H_0).
\end{aligned}
$$

With the choice $f(t)=g(t)=0$ it represents the evolution of a uniformly doped device of infinite length under a constant electric field, after a localized perturbation of the density has been created. As $t\mapsto\infty$ the solution recovers $n=c_0+2v_2\lambda^2$ and $E=V_x-\Phi_x=v_1-p_1=E_0$.

Case 2

We can look for solutions to the equation (7) of the form $u=T(t)X(x)$. In this case one obtains

$$
\begin{aligned}
&T_t=-K_mT,\\
&\alpha\mu_0U_0(2v_2\lambda^2+c_0)^{\alpha-1}X_{xx}-\mu_0(p_1-v_1)X_x+\left(K_m-\frac{\mu_0(c_0+2v_2\lambda^2)}{\lambda^2}\right)X=0
\end{aligned}
$$

where K_m is a numerical sequence which is related to the boundary and initial data.
If we set $K_m=0$ we find

$$
u(x,t)=T_0X_0(x)=A_0\ e^{-\frac{-p_1+v_1+\sqrt{\Delta_1}}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x}+B_0\ e^{-\frac{-p_1+v_1-\sqrt{\Delta_1}}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x}
$$

with $\Delta_1=(-p_1+v_1)^2+\frac{4\alpha(c_0+2v_2\lambda^2)^\alpha U_0}{\lambda^2}>0$ and T_0, A_0, B_0 constants.
The case $K_m<0$ is neglected because it leads to solutions that are not bou nded in time.
Instead by requiring

$$
K_m>\frac{\mu_0}{4\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}\left((-p_1+v_1)^2+4\alpha U_0\frac{(2v_2\lambda^2+c_0)^\alpha}{\lambda^2}\right)>0,
$$

one gets

$$
\Delta_2=\mu_0^2(-p_1+v_1)^2-4\alpha\mu_0U_0(2v_2\lambda^2+c_0)^{\alpha-1}\left(K_m-\frac{\mu_0(2v_2\lambda^2+c_0)}{\lambda^2}\right)<0
$$

and obtains the solution in the form

$$
\begin{aligned}
T_m &= T_0 e^{-K_m t},\\
X_m &= e^{\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x}\left(A_m \cos \frac{\sqrt{-\Delta_2}}{2\alpha\mu_0 U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x+\right.\\
&\qquad \left. B_m \sin \frac{\sqrt{-\Delta_2}}{2\alpha\mu_0 U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x\right),
\end{aligned}
$$

where T_0, A_m, B_m are constants to determine once the boundary and initial data are assigned.

Taking into account of the linearity of the equation and recalling that $u(t,x) = T(t)X(x)$, we get

$$
\begin{aligned}
\Phi_1(t,x) = {} & A_0\, e^{\frac{p_1-v_1-\sqrt{\Delta_1}}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x} + B_0\, e^{\frac{p_1-v_1+\sqrt{\Delta_1}}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x} +\\
& + \sum_{m=1}^{+\infty} e^{-K_m t+\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x}\left(A_m \cos \frac{\sqrt{-\Delta_2}}{2\alpha\mu_0 U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x+\right.\\
& \left. B_m \sin \frac{\sqrt{-\Delta_2}}{2\alpha\mu_0 U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x\right) + xf(t) + g(t). \qquad (11)
\end{aligned}
$$

Let suppose that the domain representing the device is the interval $[0, L]$. In order to relate A_m and B_m to the Fourier's coefficients of the initial data we set

$$
\frac{\sqrt{-\Delta_2}}{2\alpha\mu_0 U_0(2v_2\lambda^2+c_0)^{\alpha-1}} = \frac{2\pi\, m}{L},
$$

and hence

$$
K_m = \mu_0 \frac{\lambda^2(v_1-p_1)^2 + 4\alpha U_0(2v_2\lambda^2+c_0)^{\alpha}}{4\alpha\lambda^2 U_0(2v_2\lambda^2+c_0)^{\alpha-1}} + \frac{4\mu_0\pi^2 m^2 \alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}{L^2}. \qquad (12)
$$

As an example, we take the initial datum of Φ_1

$$
\Phi_1(0,x) = \begin{cases}
\frac{L}{4}\bar{E}_1 e^{\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x} & \text{if} \quad x \in \left[0, \frac{L}{4}\right[\\[2ex]
-\bar{E}_1\left(x-\frac{L}{2}\right) e^{\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x} & \text{if} \quad x \in \left[\frac{L}{4}, \frac{3L}{4}\right]\\[2ex]
-\frac{L}{4}\bar{E}_1 e^{\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x} & \text{if} \quad x \in \left]\frac{3L}{4}, L\right],
\end{cases}
$$

where $\bar{E}_1 = \text{const.} \neq 0$, regularized according to theory of Fourier's series. Since the Fourier expansion of $e^{-\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x}\Phi_1(0,x)$ is

$$
\sum_{m=1}^{+\infty} B_m \sin \frac{2\pi\, m}{L}x,
$$

provided $f(0) = g(0) = 0$, one has $A_0 = B_0 = 0$. Therefore we have the s olution

$$
\Phi_1(t,x) = xf(t) + g(t) + \sum_{m=1}^{+\infty} e^{-K_m t+\frac{p_1-v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}x} B_m \sin \frac{2\pi\, m}{L}x
$$

where the coefficients
B_m are given by

$$B_m = \frac{L\bar{E}_1\left(\sin\left(\frac{\pi m}{2}\right) - \sin\left(\ frac3\pi m2\right) + \pi m\right)}{2\pi^2 m^2}. \tag{13}$$

With the choice $f(t) = g(t) = e^{-at}\sin\omega_1 t$, with $a > 0$ and ω_1 constants, the resulting solution solves the boundary value problem with boundary conditions for the perturbation of density and electric potential given by

$$\begin{aligned}
n_1(t,0) &= \lambda^2 \sum_{m=1}^{+\infty} e^{-K_m t} \frac{2\pi m(p_1 - v_1)B_m}{\alpha U_0(2v_2\lambda^2 + c_0)^{\alpha-1}L},\\
n_1(t,L) &= \lambda^2 \sum_{m=1}^{+\infty} e^{-K_m t + \frac{p_1 - v_1}{2\alpha U_0(2v_2\lambda^2+c_0)^{\alpha-1}}L} \frac{2\pi m(p_1 - v_1)B_m}{\alpha U_0(2v_2\lambda^2 + c_0)^{\alpha-1}L},\\
\Phi_1(t,0) &= e^{-at}\sin\omega_1 t, \quad \Phi_1(t,L) = (L+1)e^{-at}\sin\omega_1 t.
\end{aligned}$$

We remark that the previous series are convergent $\forall t > 0$ because K_m is quadratic in m (see (12)).

As $t \mapsto +\infty$ one has $n_1(t,0) = n_1(t,L) = \Phi_1(t,0) = \Phi_1(t,L) = 0$.

In the particular case $p_1 = v_1$ the initial datum becomes

$$\Phi_1(0,x) = \begin{cases} \frac{L}{4}\bar{E}_1 & \text{if} \quad x \in \left[0, \frac{L}{4}\right[\\ -\bar{E}_1\left(x - \frac{L}{2}\right) & \text{if} \quad x \in \left[\frac{L}{4}, \frac{3L}{4}\right]. \\ -\frac{L}{4}\bar{E}_1 & \text{if} \quad x \in \left]\frac{3L}{4}, L\right]. \end{cases}$$

The relation $n_1(t,x) = \lambda^2\Phi_{1xx}(t,x)$ implies that the init ial datum for n_1 is given by the localized perturbation

$$n_1(0,x) = -\lambda^2\left[\delta\left(x - \frac{L}{4}\right) - \delta\left(x - \frac{3L}{4}\right)\right]\bar{E}_1$$

with δ the distribution Dirac's delta.

In this case the solution reads

$$\Phi_1(t,x) = xf(t) + g(t) + \sum_{m=1}^{+\infty} e^{-K_m t} B_m \sin\frac{2\pi\, m}{L}x$$

where the coefficients
B_m are given by (13).
With the previous choice for $f(t) = g(t)$,
the resulting solution solves the boundary value problem with boundary conditions for the perturbation of density and electric potential given by

$$n_1(t,0) = 0, \qquad n_1(t,L) = 0,$$
$$\Phi_1(t,0) = e^{-at}\sin\omega_1 t, \quad \Phi_1(t,L) = (L+1)e^{-at}\sin\omega_1 t$$

As $t \mapsto +\infty$
one has $\Phi_1(t,0) = \Phi_1(t,L) = 0$.
Always in the case $p_1 = v_1$ if we take
the initial value for Φ_1

$$\Phi_1(0,x) = x^2 - \frac{2L}{3}x \qquad \text{if} \quad x \in [0,\, L]$$

the initial value for n_1 is given by

$$n_1(0,x) = 2\lambda^2.$$

Since $\Phi_1(0,x)$ has a Fourier expansion given by

$$\Phi_1(0,x) = \sum_{m=1}^{+\infty}\left(A_m \cos\frac{2\pi\, m}{L}x + B_m \sin\frac{2\pi\, m}{L}x\right),$$

provided $f(0) = g(0) = 0$, one has $A_0 = B_0 = 0$. Therefore we have the solution

$$\Phi_1(t,x) = xf(t) + g(t) + \sum_{m=1}^{+\infty} e^{-K_m t}\left(A_m \cos\frac{2\pi\, m}{L}x + B_m \sin\frac{2\pi\, m}{L}x\right)$$

where the coefficients
A_m and B_m are given by

$$A_m = \frac{L^2}{\pi^2 m^2}, \quad B_m = -\frac{L^2}{3\pi m}. \tag{14}$$

Again with the choice $f(t) = g(t) = e^{-at}\sin\omega_1 t$, with $a > 0$ and ω_1 constants,
the resulting solution solves the boundary value problem with boundary conditions

$$n_1(t,0) = n_1(t,L) = \lambda^2 \sum_{m=1}^{+\infty}\left(-4e^{-K_m t}\right),$$
$$\Phi_1(t,0) = \sum_{m=1}^{+\infty}\left(\frac{e^{-K_m t}L^2}{\pi^2 m^2}\right) + e^{-at}\sin\omega_1 t,$$
$$\Phi_1(t,L) = \sum_{m=1}^{+\infty}\left(\frac{e^{-K_m t}L^2}{\pi^2 m^2}\right) + (L+1)e^{-at}\sin\omega_1 t,$$

As $t \mapsto +\infty$ one has $n_1(t,0) = n_1(t,L) = \Phi_1(t,0) = \Phi_1(t,L) = 0$.
If $a = 0$, for $t >> 1$, $\Phi_1(t,x) \approx (x+1)\sin\omega_1 t$ which gives a time dependent electric field $E(t,x) = -H_0 \sin\omega_1 t$.

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Nonlinear Propagation and Stability in Thermodinamycal Processes of Con tinuous Media.).

REFERENCES

1. S. Selberherr, 1984, Analysis and Simulation of Semiconductor Devices, Springer, Wien, New York.
2. W. Hänsch, 1991, The Drift-Diffusion Equation and its Applications in MOSFET modeling, Springer, Wien.
3. P. Markowich, C. A. Ringhofer, C. Schmeiser, 1990, Semiconductor Equations, Springer, Wien.
4. M. Loffreo and L. Morato, Lagrangian variational principle in stochastic mechanics: gauge structure and stability, *J. Math. Phys.* **30** 354 (1989).
5. M. Loffreo and L. Morato, On the creation of quantum vortex lines in rotating HeII, *Il Nuovo Cimento* **108B** 205 (1993)
6. M. G. Ancona and H. F. Tiersten, Macroscopic physics of the silicon inversion layer, *Phys. Rew. B* **35**, 7959 (1987).
7. M. G. Ancona and G. J. Iafrate, Quantum correction to the equation of state of an electron electron gas in a semiconductor, *Phys. Rew. B* **39**, 9536 (1989).
8. A.M. Anile, G. Mascali, V.Romano. Recent developments in hydrodynamical mode ling of semiconductors (2003) 1:54 in *Mathematical Problems in Semiconductor Physics*, Lecture Notes in Mathematics 1832, Springer (2003).
9. P. Degond and C. Ringhofer, Quantum moment hydrodynamics and the entropy principle, *J. Stat. Physics* **118**, 625 (2005).
10. A. Jüngel, 2001, Quasi-Hydrodynamics Semiconductor Equations, Birkhäuser, Basel.
11. R. Pinnau, Uniform convergence of an exponentially fitted scheme for the quantum drfit diffusion model, *SIAM J. Numer. Anal.* **42** 1648 (2004).
12. C. de Falco, E. Gatti, A. L. Lacaita, R. Sacco, Quantum-co rrected drift-diffusion models for transport in semiconductors, *J. Comp. Physics* **204** 533 (2004).
13. N. Ben Abdallah and A. Unterreiter, On the stationary qua ntum drift-diffusion model,*Z. Angew. Math. Physik* **49** 251 (1998).

14. R. Romano, M. Torrisi, R. Tracinà, Approximate solutions to the quantum d rift-diffusion model of
semiconductors, *J. Math. Physics*, **48** 1, 23501 (2007).
15. M. Torrisi, R. Tracinà, Second-order differential invariants of
a family of diffusion equations, *J. Phys. A: Math. Gen.*,
38, pp 7519-7526 (2005).