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# HOLE MOBILITIES IN THE SILICON SEMICONDUCTOR : THE CASE OF WARPED BAND

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## Abstract.

The maximum entropy principle is used to get a consistent hydrodynamical model for the transport of holes in semiconductors and then from this asymptotic energy-transport and drift-diffusion models are derived. The valence bands are described with warped functions. An explicit formula for the low field mobility is deduced.

*Keywords*: hole transport; hydrodynamical, energy transport and drift-diffusion models.

## 1. Introduction

Following the same approach presented in[1,3,4,2] for electrons a hydrodynamical model for hole transport in silicon semiconductors based on the maximum entropy principle (hereafter MEP) is presented. A similar model had been already formulated in[5] but taking into account only a single parabolic band. Here both heavy and light holes are considered in the warped approximation along with the split-off band. Scattering with non-polar optical phonons and acoustic phonons are considered. Due to the anisotropy the bands the determination of the constitutive equations is rather involved and suitable expansions must be introduced to make the problem analytically tractable.

The 8-moment case is investigated in detail and from this limiting energy-transport and drift-diffusion models are deduced under appropriate asymptotic scalings. An explicit expression for the low field mobility is obtained.

## 2. The kinetic semiclassical model

The hole energy spectrum in Si is represented by three bands.[6] A schematic representation is given in fig. 1. The first two bands are the heavy and light valence bands which are degenerate at $\mathbf{k} = 0$, where they reach their maximum. The third one is the so-called *split-off band* which is separated from the first two by the spin-orbit energy $\Delta = 0.0443$ eV at $\mathbf{k} = 0$. Because of its low density of states and its energy separation the split-off valence band is usually neglected.

The energy bands of heavy and light holes are analytically approximated with *warped bands*

$$\mathcal{E}_H(\mathbf{k}) = \frac{\hbar^2}{2m_e}\left\{Ak^2 \mp \left[B^2k^4 + C^2(k_x^2k_y^2 + k_y^2k_z^2 + k_z^2k_x^2)\right]^{1/2}\right\}, \tag{1}$$

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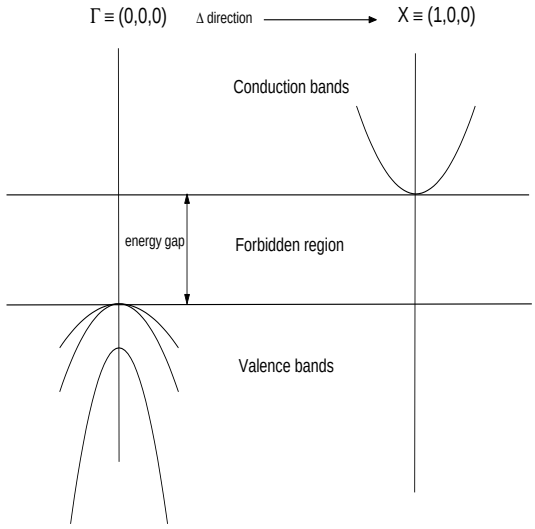


Fig. 1. A schematic representation of the energy conduction and valence bands ($\mathcal{E}(\mathbf{k})$ versus $\mathbf{k}$ in arbitrary units) in Si. The conduction bands for holes are obtained from those of valence by reversing the sign.

where $H = +, -$ stand for the light and heavy hole bands respectively. $k_x, k_y, k_z$ are the component of $\mathbf{k}$ with respect to the principal crystallographic axes. The parameters $A$, $B$ and $C$ depend on the specific material. The constant energy surface have a warped form (see figure 2).

The (microscopic) hole velocity $\mathbf{v}$ in the heavy and light warped bands is obtained with the quantum mechanics formula $v^i = \frac{1}{\hbar}\nabla\mathcal{E}$ and reads

$$v_i = \frac{\hbar}{2m_e} k_i \left\{ 2\,A \mp \frac{2k^2B^2 + C^2k_\perp^2}{\sqrt{B^2k^4 + C^2(k_x^2k_y^2 + k_y^2k_z^2 + k_z^2k_x^2)}} \right\}$$

where $\mathbf{k}_\perp$ is the component of $\mathbf{k}$ orthogonal to the i-th crystallographic axis.

A simple properties, useful in the sequel, is the following.

**Proposition 1.** *The warped energy bands have the same (discrete) symmetries of the cube, in particular the permutation of axes.*

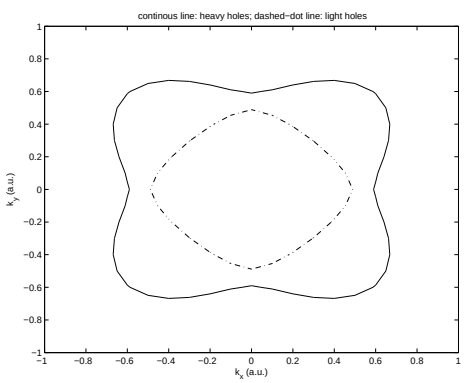


Fig. 2. Constant energy surface of the warped bands at $k_z = 0$ .

The semiclassical description of hole transport in semiconductors consists of a transport equation for each band coupled to the Poisson equation for the electric potential

$$\frac{\partial f_H}{\partial t} + v^i(\mathbf{k})\frac{\partial f_H}{\partial x^i} + \frac{eE^i}{\hbar}\frac{\partial f_H}{\partial k^i} = \mathcal{C}[f_H], \tag{2}$$

$$E_i = -\frac{\partial \phi}{\partial x_i}, \tag{3}$$

$$\epsilon \Delta \phi = -e(N_D - N_A - n + p), \tag{4}$$

where $\epsilon$ is the dielectric constant $n$, $p$ $N_D$, $N_A$ the electron, hole, acceptor and donor densities. $e$ is the absolute value of the elementary charge. The indexes H can be + (light holes) , - (heavy holes).

$\mathcal{C}[f_H]$ comprises intraband acoustic and non polar optical in an additive way. In the linear approximation each of them is written as

$$\mathcal{C}[f_H] = \int d\mathbf{k}' \left[P(\mathbf{k}', \mathbf{k}) f' - P(\mathbf{k}, \mathbf{k}') f\right]$$

with $P(\mathbf{k}, \mathbf{k}')$ transition rate from the state with wave-vector $\mathbf{k}$ to the state with wave-vector $\mathbf{k}'$. In the sequel we will make use of the detailed balance principle which for intraband transition reads

$$P(\mathbf{k}', \mathbf{k}) = \exp\left[-\frac{(\mathcal{E} - \mathcal{E}')}{k_B\, T_L}\right] P(\mathbf{k}, \mathbf{k}'). \tag{5}$$

In detail the scattering mechanisms are taken to have the following transition rates.[7,8]

- *intraband non-polar optical phonon scattering*

$$P(\mathbf{k}, \mathbf{k}') = \mathcal{K}_{op} \begin{bmatrix} N_{op} \\ N_{op} + 1 \end{bmatrix} \delta[\mathcal{E}_c(\mathbf{k}') - \mathcal{E}_c(\mathbf{k}) \mp \hbar\omega_{op}]$$

  where $\delta$ is Dirac's delta, $\mathcal{K}_{op} = \frac{(D_t K)^2}{8\,\pi^2\,\rho\,\omega_{op}}$ a coupling constant and $N_{op}$ the optical phonon distribution at equilibrium

$$N_{op} = \frac{1}{\exp(\hbar\omega_{op}/k_B T_L) - 1}, \tag{6}$$

- *intraband acoustic phonon scattering*

$$P(\mathbf{k}, \mathbf{k}') = \mathcal{K}_{ac}\; q \begin{bmatrix} N_q \\ N_q + 1 \end{bmatrix} \frac{1}{4}(1 + 3\,(\mathbf{n}\cdot\mathbf{n}')^2)\delta(\mathcal{E}_c(\mathbf{k}') - \mathcal{E}_c(\mathbf{k}) \mp \hbar q v_s),$$

  where the acoustic phonon wave vector is given by

$$q = \sqrt{2}k\sqrt{1 - \mathbf{n}\cdot\mathbf{n}'},$$

  $\mathcal{K}_{ac} = \frac{\Xi_d^2}{8\pi^2\rho v_s}$, with $\mathbf{n} = \mathbf{k}/k$, $\mathbf{n}' = \mathbf{k}'/k'$, $v_s$ longitudinal component of the sound speed.

The value of the physical parameters can be found in ref..[5]

## 3. Macroscopic balance equations

Starting from the Boltzmann equation (2), it is possible to obtain macroscopic equations for the holes multiplying equation (2) by a weight function $\psi = \psi(\mathbf{k})$ and integrating with respect to $\mathbf{k}$ over $\mathbb{R}^3$. If one indicates with $f_H$ the hole distribution in one of the bands and sets

$$M_\psi = \int_{\mathbb{R}^3} \psi(\mathbf{k}) f_H(\mathbf{x}, \mathbf{k}, t) d\mathbf{k},$$

which is the moment of $f_H$ relative to the weight function $\psi(\mathbf{k})$, the following equation

$$\frac{\partial M_\psi}{\partial t} + \int_{\mathbb{R}^3} \psi(\mathbf{k})\, \mathbf{v} \cdot \nabla_{\mathbf{x}} f_H d\mathbf{k} + \frac{e\mathbf{E}}{\hbar} \int_{\mathbb{R}^3} \psi(\mathbf{k}) \nabla_{\mathbf{k}} f_H d\mathbf{k} = \int_{\mathbb{R}^3} \psi(\mathbf{k})\ \mathcal{C}[f_H] d\mathbf{k} \tag{1}$$

is obtained. We set the weight function $\psi(\mathbf{k})$ equal to 1, $\hbar k^i$, $\mathcal{E}(\mathbf{k})$, $\mathcal{E}(\mathbf{k}) v^j$, $i, j = 1, 2, 3$ getting the following system of balance equations for density, average crystal momentum, energy and energy flux for each population of holes

$$\frac{\partial p_H}{\partial t} + \frac{\partial (p_H\ V_H^i)}{\partial x^i} = p_H\ C_p, \tag{2}$$

$$\frac{\partial (p_H\ P_H^j)}{\partial t} + \frac{\partial (p_H\ U_H^{ij})}{\partial x^j} - p_H\ e\ E^j = p_H\ C_{P_H}^j, \qquad j = 1, 2, 3, \tag{3}$$

$$\frac{\partial p_H\ W_H}{\partial t} + \frac{\partial (p_H\ S_H^i)}{\partial x^i} - p_H\ e\ E_i V_H^i = p_H\ C_{W_H}, \tag{4}$$

$$\frac{\partial (p_H\ S_H^j)}{\partial t} + \frac{\partial (p_H\ F_H^{ij})}{\partial x^i} - p_H\ e\ E_i G_H^{ji} = p_H\ C_{S_H}^j \qquad j = 1, 2, 3. \tag{5}$$

where

$p_H = \int_{\mathbb{R}^3} f_H d\mathbf{k}$ is the hole density,

$V_H^i = \frac{1}{p_H} \int_B v^i f_H d\mathbf{k}$ is the average hole velocity,

$C_p = \frac{1}{p_H} \int_{\mathbb{R}^3} \mathcal{C}[f_H] d\mathbf{k}$ is the density production,

$P_H^j = \frac{1}{p_H} \int_{\mathbb{R}^3} \hbar k^j f_H d\mathbf{k} = m_H^* V_H^j \qquad j = 1, 2, 3$ is the average crystal momentum,

$U_H^{ij} = \frac{1}{p_H} \int_{\mathbb{R}^3} f_H v^i \hbar k^j d\mathbf{k}$ is the crystal momentum flux,

$C_{P_H}^j = \frac{1}{p_H} \int_{\mathbb{R}^3} \hbar k^j \mathcal{C}[f_H](\mathbf{x}, \mathbf{k}, t) d\mathbf{k}$ is the average crystal momentum production,

$W_H = \frac{1}{p_H} \int_{\mathbb{R}^3} \mathcal{E}(\mathbf{k}) f_H d\mathbf{k}$ is the average hole energy,

$S_H^i = \frac{1}{p_H} \int_{\mathbb{R}^3} \mathcal{E}(\mathbf{k}) f_H v^i d\mathbf{k}$ is the energy flux,

$C_{W_H} = \frac{1}{p_H} \int_{\mathbb{R}^3} \mathcal{E}(\mathbf{k}) \mathcal{C}[f_H](\mathbf{x}, \mathbf{k}, t) d\mathbf{k}$ is the energy production,

$$F_H^{ij} = \frac{1}{p_H}\int_{\mathbb{R}^3} \mathcal{E}(\mathbf{k}) v^i v^j f d\mathbf{k} \qquad \text{is the flux of energy flux,}$$
$$G_H^{ij} = \frac{1}{p_H}\int_{\mathbb{R}^3} \frac{1}{\hbar} f_H \frac{\partial(\mathcal{E} v^i)}{\partial k^j} d\mathbf{k},$$
$$C_{S_H}^j = \frac{1}{p_H}\int_{\mathbb{R}^3} \mathcal{E}(\mathbf{k}) v^j \mathcal{C}[f_H] d\mathbf{k} \qquad \text{is the flux energy production,}$$

with

$$m_H^* = \frac{J3}{J2} m_e.$$

$J_2$ and $J_3$ are given by

$$J_2 = \int_{S^2} \frac{T^2}{D^{3/2}} \cos^2 \vartheta \, d\Omega,$$
$$J_3 = 2\int_{S^2} \frac{T}{D^2} \cos^2 \vartheta \, d\Omega,$$

with $S^2$ the unit sphere of $\mathbb{R}^3$, $d\Omega = \sin\vartheta \, d\vartheta \, d\varphi$ element of solid angle and

$$D = A \mp \sqrt{B^2 + C^2 \sin^2\vartheta(\sin^2\vartheta \sin^2\varphi \cos^2\varphi + \cos^2\vartheta)}$$

$$T = \left(2A \mp \frac{2B^2 + C^2 \sin^2\vartheta}{\sqrt{B^2 + C^2 \sin^2\vartheta(\sin^2\vartheta \sin^2\varphi \cos^2\varphi + \cos^2\vartheta)}}\right) \frac{1}{\sqrt{D}}.$$

$p_H$, $\mathbf{V}_H$,$W_H$ and $\mathbf{S}_H$ are assumed as fundamental variables since they have a direct physical meaning. Therefore there is the problem of closing the system (2)-(5) by expressing the fluxes $U_H^{ij}$, $F_H^{ij}$, $G_H^{ij}$ and the production term $C_p$, $C_{P_H}^j$, $C_{W_H}$, $C_{S_H}^j$ as functions of $p_H$, $V_H^i$,$W_H$ and $S_H^i$

## 4. Maximum entropy principle and closure relations

In order to get the closure relations for semiconductor hydrodynamical models, many strategies have been proposed, often without any consistent mathematical or physical rationale.[9] In,[3,4,2] the MEP[10,11,12,13] has been used to solve the problem of finding self-consistent closure relations for electron macroscopic balance equations, both for Si and GaAs. Here we employ MEP to get the required closure relations for the system (2)-(5).

According to this principle, if we have a finite number of known moments for each band H

$$M_{H,\alpha} = \int_{\mathbb{R}^3} \psi_\alpha f_H d\mathbf{k}, \quad H = -, +,$$

then the distribution function $f_H^{ME}$, which can be used for an evaluation of the unknown moments, corresponds to the extremum of the entropy functional, under the restrictions

$$M_{H,\alpha} = \int_{\mathbb{R}^3} \psi_\alpha f_H^{ME} d\mathbf{k}. \tag{1}$$

The formal setting of the MEP has been developed in the framework of the information theory by Shannon and applied for the first time to statistical mechanics by Jaynes.[10] He showed that many questions of classical and quantum mechanics can be reformulated as statistical inference problems where the MEP distribution represents the least biased distribution with respect to the only knowledge of a finite number of moments.

In the case of a sufficiently dilute hole gas the entropy functional, according to the classical limit of the expression arising in the Fermi statistics, is for each population

$$-k_B \int_{\mathbb{R}^3} (f_H \log f_H - f_H) d\mathbf{k} \tag{2}$$

while the total entropy reads

$$s = -k_B \sum_H \int_{\mathbb{R}^3} (f_H \log f_H - f_H) d\mathbf{k_H}. \tag{3}$$

By introducing the Lagrangian multipliers $\Lambda_{H,\alpha}$, looking for the extremals of the entropy is equivalent to looking for the extremals without constraints of the following functional

$$s' = \sum_{H,\alpha} \Lambda_{H,\alpha} \left( \int_{\mathbb{R}^3} \psi_{H,\alpha} f_H d\mathbf{k} - M_{H,\alpha} \right) - s \tag{4}$$

which is the Legendre transform of the entropy functional $s$. From variational calculus,

$$\delta s' = k_B \sum_H \int_{\mathbb{R}^3} \delta f_H \log f_H d\mathbf{k} + \sum_{H,\alpha} \Lambda_{H,\alpha} \int_{\mathbb{R}^3} \psi_{H,\alpha} \delta f_H d\mathbf{k} = 0$$

for arbitrary $\delta f_H$. Therefore for each band H

$$\log f_H + \frac{\Lambda_{H,\alpha} \psi_{H,\alpha}}{k_B} = 0$$

from which we get the maximum entropy distribution function as

$$f_H^{ME} = \exp \left( - \sum_\alpha \frac{\Lambda_{H,\alpha} \psi_{H,\alpha}}{k_B} \right). \tag{5}$$

If we make the choice of the weight functions $\psi_{H,\alpha} = (1, \mathbf{v}, \mathcal{E}, \mathcal{E}\mathbf{v})$ for each band H=+,-, one has to introduce the Lagrangian multipliers $\Lambda_H = (\lambda_H, \boldsymbol{\lambda}_H^P, \lambda_H^W, \boldsymbol{\lambda}_H^S)$ and the maximum entropy distribution function reads

$$f_H^{ME} = \exp \left[ - \left( \frac{1}{k_B} \lambda_H + \boldsymbol{\lambda}_H^P \cdot \mathbf{v} + \lambda_H^W \mathcal{E} + \boldsymbol{\lambda}_H^S \cdot \mathbf{v}\, \mathcal{E} \right) \right]. \tag{6}$$

In order to complete the program, it is necessary to express the Lagrangian multipliers in terms of the fundamental variables by evaluating the constraints (1). On account of the high nonlinearities, we were not able to find out an analytical explicit form of the multipliers, but, by proceeding as in,[3] we expand $f_H^{ME}$ with respect to a parameter of anisotropy $\delta$ and solve the resulting equations at several orders in such a parameter. In particular with the previous choice of the weights in the moments, one expand $f_H^{ME}$ as

$$\begin{aligned} f_H^{ME} &= \exp \left[ - \left( \frac{1}{k_B} \lambda_H + \lambda_H^W \mathcal{E} + \delta \boldsymbol{\lambda}_H^P \cdot \mathbf{v} + \delta \boldsymbol{\lambda}_H^S \cdot \mathbf{v}\, \mathcal{E} \right) \right] = \\ &\exp \left( - \frac{1}{k_B} \lambda_H - \lambda^W \mathcal{E} \right) \left[ 1 - \delta \left( \boldsymbol{\lambda}_H^P \cdot \mathbf{v} + \boldsymbol{\lambda}_H^S \cdot \mathbf{v} \mathcal{E} \right) \right] + o(\delta) \end{aligned} \tag{7}$$

Evaluating the constraint of the energy, one gets

$$\lambda^W = \frac{3}{2\,W}$$

which is the same result obtained in the parabolic case. Concerning the other multipliers, from the representation theorem we have

$$\boldsymbol{\lambda}^V = b_{11}(W)\mathbf{V} + b_{12}(W)\mathbf{S} \tag{8}$$

$$\boldsymbol{\lambda}^S = b_{21}(W)\mathbf{V} + b_{22}(W)\mathbf{S} \tag{9}$$

and evaluating the constraints (3) and (5) one finds

$$b_{11} = -\frac{7\,m_e}{W}\,\frac{J_1}{J_2}, \quad b_{12} = b_{21} = \frac{3\,m_e}{W^2}\,\frac{J_1}{J_2}, \quad b_{22} = -\frac{9\,m_e}{5W^3}\,\frac{J_1}{J_2} \tag{10}$$

where

$$J_1 = \int_{S^2} D^{-3/2} d\Omega$$

The lagrangian multiplier relative to the density is given by

$$\lambda = -k_B\,\log\frac{\hbar^3 p}{2J_1\pi^{1/2}(m_e W/3)^{3/2}}.$$

Once $f_{ME}$ has been determined, the needed closure relations are obtained by inserting $f_{ME}$ in the kinetic definition of the unknown fluxes and production terms, e.g. for the fluxes up to first order in $\delta$ one has

$$U^{ij} = U(W)\,\delta^{ij}, \quad F^{ij} = F(W)\,\delta^{ij}, \quad G^{ij} = G(W)\,\delta^{ij},$$

where

$$U(W) = \frac{2}{3}\,W \text{ (as in the parabolic case)}, \quad F(W) = \frac{5}{6m_e}\,\frac{J_2}{J_1}\,W^2, \tag{11}$$

$$G(W) = \frac{1}{2m_e}\,\frac{J_4}{J_1}\,W, \tag{12}$$

with

$$J_4 = \int_{S^2}\left[\frac{\cos^2\vartheta}{D^{3/2}}\frac{(2A - T\sqrt{D})^2 \mp 4B^2}{\sqrt{B^2 + C^2\,\sin^2\vartheta(\sin^2\vartheta\,\sin^2\varphi\,\cos^2\varphi + \cos^2\vartheta)}} + \frac{T}{D} + \frac{T^2}{D^{3/2}}\cos^2\vartheta\right] d\Omega.$$

## 5. The energy transport and drift diffusion-limiting model

First we rewrite the hydrodynamical model for holes in the form

$$\frac{\partial p_H}{\partial t} + \frac{\partial(p_H\,V_H^i)}{\partial x^i} = 0, \tag{1}$$

$$\frac{\partial(p_H P_H^j)}{\partial t} + \frac{\partial(p_H U_H^{ij})}{\partial x^j} - e p_H E^j = p_H\left[(c_{11}(W_H)V_H^i + c_{12}(W_H)S_H^i\right], \tag{2}$$

$$\frac{\partial p_H W_H}{\partial t} + \frac{\partial(p_H\,S_H^i)}{\partial x^i} - e\,p_H\,E^i V_H^i = -p_H\frac{W_H - W_0}{\tau_{W_H}}, \tag{3}$$

$$\frac{\partial(p_H S_H^j)}{\partial t} + \frac{\partial(p_H F_H^{ij})}{\partial x^i} - e p_H E^i G_H^{ji} = p_H\left[c_{21}(W_H)V_H^i + c_{22}(W_H)S_H^i\right] \tag{4}$$

with $W_0 = 3/2k_BT_L$ and with an obvious meaning for $\tau_{W_H}$ (the energy relaxation time), $c_{11}(W_H)$, $c_{12}(W_H)$, $c_{21}(W_H)$, $c_{22}(W_H)$.

As in[14] let us assume the following scaling

$$t = \mathcal{O}\left(\frac{1}{\delta^2}\right), \tau_W = \mathcal{O}\left(\frac{1}{\delta^2}\right), x^i = \mathcal{O}\left(\frac{1}{\delta}\right), \mathbf{V}_H = \mathcal{O}(\delta), \mathbf{S}_H = \mathcal{O}(\delta). \tag{5}$$

The first condition is a long time scaling, that is almost stationary regime. The second one means that the energy relaxation time must be sufficiently long with respect to the typical time of the transient. $(5)_3$ is the typical diffusion scaling, while $(5)_4$,$(5)_5$ are consistent with the expansion made to get the closure relations. Under the conditions (5), equating to zero at the various order in $\delta$ the terms appearing in the balance equations one gets the following compatibility conditions

$$\frac{\partial p_H}{\partial t} + \frac{\partial (p_H V_H^i)}{\partial x^i} = 0, \tag{6}$$

$$\frac{\partial (p_H V_H^i)}{\partial t} = 0, \quad \frac{\partial (p_H\ S_H^i)}{\partial t} = 0, \tag{7}$$

$$\frac{\partial (p_H\ U_H^{ij})}{\partial x^j} - e\, p_H\ E^i - c_{11} p_H\ V_H^i - c_{12}\ p_H\ S_H^i = 0, \tag{8}$$

$$\frac{\partial (p_H W_H)}{\partial t} + \frac{\partial (p_H\ S_H^i)}{\partial x^i} - e\, p_H\, E^i V_H^i + p_H \frac{W_H - W_0}{\tau_{W_H}} = 0, \tag{9}$$

$$\frac{\partial (p_H F_H^{ij})}{\partial x^i} - e\, p_H\, E^i G_H^{ij} - c_{21}\ p_H\ V_H^i - c_{22}\ p_H\ S_H^i = 0. \tag{10}$$

Equations (8) and (10) are a linear system for $\mathbf{V}_H$ and $\mathbf{S}_H$ whose solution is

$$\mathbf{V}_H = D_{11}(W_H)\nabla \log p_H + D_{12}(W_H)\nabla W_H + D_{13}(W_H)\nabla\phi, \tag{11}$$

$$\mathbf{S}_H = D_{21}(W_H)\nabla \log p_H + D_{22}(W_H)\nabla W_H + D_{23}(W_H)\nabla\phi. \tag{12}$$

The elements of the diffusion matrix $D = (D_{ij})$ read

$$D_{11} = \frac{\frac{2}{3}c_{22}W_H - \frac{5J_2}{6J_1}c_{12}\frac{W_H^2}{m_e}}{c_{11}c_{22} - c_{12}c_{21}}, \quad D_{12} = \frac{\frac{2}{3}c_{22} - \frac{10J_2}{6J_1}c_{12}\frac{W_H}{m_e}}{c_{11}c_{22} - c_{12}c_{21}}$$

$$D_{13} = e\frac{c_{22} - \frac{J_4}{2J_1}c_{12}\frac{W_H}{m_e}}{c_{11}c_{22} - c_{12}c_{21}}, \quad D_{21} = \frac{\frac{5J_2}{6J_1}c_{11}\frac{W_H^2}{m_e} - \frac{2}{3}c_{21}W_H}{c_{11}c_{22} - c_{12}c_{21}},$$

$$D_{22} = \frac{\frac{10J_2}{6J_1}c_{11}\frac{W_H}{m_e} - \frac{2}{3}c_{21}}{c_{11}c_{22} - c_{12}c_{21}}, \quad D_{23} = -e\frac{c_{21} - \frac{J_4}{2J_1}c_{11}\frac{W_H}{m_e}}{c_{11}c_{22} - c_{12}c_{21}}.$$

The balance equations for density and energy (6) and (10) closed with the relations (11),(12) are the energy-transport model for holes based on MEP. From this latter a drift-diffusion model is obtained as isothermal limit formally setting $\tau_{W_H} \mapsto 0$

$$\mathbf{J}_H = p_H \mathbf{V}_H = D_{11}(W_0)\nabla p_H + p_H D_{13}(W_0)\nabla\phi, \tag{13}$$

$$\frac{\partial p_H}{\partial t} + \nabla \cdot \mathbf{J}_H = 0. \tag{14}$$

By comparing (13) with the expression of $\mathbf{J}$ in the form

$$\mathbf{J} = -D_p \nabla p_H - \mu_{p0} p_H \nabla\phi,$$

one can identify the diffusivity coefficient $D_p$ and the low field mobility $\mu_{p0}$ as

$$D_p = -D_{11}(W_0), \quad \mu_{p0} = -D_{13}(W_0). \tag{15}$$

One observes that

$$D_p = \mu_{p0}\frac{2W_0}{3e} = \mu_{p0}\frac{k_B T_L}{e} \tag{16}$$

which is the Einstein relation.

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