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Simulation of the dynamics of an olympic rowing boat

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Abstract.

A tool for the prediction of the performance of olympic rowing boats is presented and discussed. The equations of motion include the full dynamics of the boat, oars and oarsmen and are obtained by modelling the rowers motion, oar forces and fluid-structure intercation forces.

The proposed algorithm is implemented in a C++ code which has proved to produce consistent results for any crew configuration tested.

Keywords: Rowing, Dynamical System, Fluid Dynamics, Fluid-Structure Interaction

1. Introduction

The dynamics of rowing skulls is rather complex. The periodic forces imposed at the oars as well as those generated by the movement of the rowers induce *secondary motions*, superimposed to the main forward motion, which dissipate energy that would have a better use thrusting the boat forward. Indeed, secondary motions may account for a significant part of total energy spent by the rowers.

This aspect is often neglected in the common design studies, which typically concentrate on the mean motion alone, hence a more in-depth analysis could grant significant improvements in the boat efficiency.

A full dynamic model requires to account not only for the thrust force at the oarlocks and the fluid-dynamic forces, but also for the inertial effects produced by the motion of the rowers. It is a rather complex fluid-structure interaction problem which can be tackled with different approaches. A direct approach would integrate the boat dynamics to a full Navier-Stokes free surface solver. Yet, this procedure is rather costly in terms of computing time. In this work we present an alternative based on simulating the energy dissipation due to the secondary motions by means of a potential problem. A full Navier-Stokes free surface model is used only to estimate the resistances due to the mean motion.

The dynamics of rowing has received a certain attention in the last years, for instance by A. Dudhia,[2] W.C. Atkinson,[1] M. van Holst[6] and L. Lazauskas,[4] although those studies have been reported mainly on web sites and very little peer reviewed literature is available. Furthermore, none of these works considers the full dynamics of the boat, being limited to the analysis of the effects of the horizontal acceleration.

We have chosen instead to consider the complete skull dynamics in the symmetry plane, thus including also pitching and vertical movements. The computation of the inertial

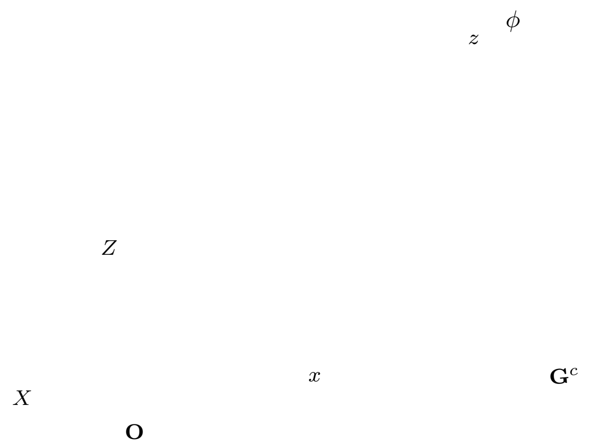

Fig. 1. The global and relative reference frames

We also introduce a relative reference frame $(\mathbf{G}_c; x, y, z)$, attached to the boat hull, which is supposed to be a rigid body, and centered in its baricenter $\mathbf{G}_c$. The versors associated to the x, y and z axes in this frame of reference will be indicated by $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$, respectively, (see Fig 1).

With these assumptions, the pitch angle ϕ is the angle between $\mathbf{e}_X$ and $\mathbf{e}_x$, and is positive when the bow lowers w.r.t the horizontal line. Once we have introduced the rotation matrix

$$\mathcal{R}(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}, \tag{1}$$

we can write the coordinate transformation law for a generic point P as

$$\begin{bmatrix} R_X^P \\ R_Y^P \\ R_Z^P \end{bmatrix} = \mathcal{R}^T(\phi) \begin{bmatrix} r_x^P \\ r_y^P \\ r_z^P \end{bmatrix} + \begin{bmatrix} G_X^c \\ G_Y^c \\ G_Z^c, \end{bmatrix} \tag{2}$$

where positions in the global system are denoted by capital letters.

Transformations between velocity and acceleration vectors in the two reference frames assume the form

$$\mathbf{V}^P = \dot{\mathbf{P}} = \mathbf{v}^P + \dot{\mathbf{G}}^C + \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{G}^C) \tag{3}$$

and

$$\mathbf{A}^P = \ddot{\mathbf{P}} = \mathbf{a}^P + \ddot{\mathbf{G}}^C + \dot{\boldsymbol{\omega}} \times (\mathbf{P} - \mathbf{G}^C) + \boldsymbol{\omega} \times \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{G}^C) + 2\boldsymbol{\omega} \times \mathbf{v}^P, \tag{4}$$

being $\boldsymbol{\omega} = \dot{\phi}\, \mathbf{e}_Y$ the angular velocity vector. Here, the dot symbol denotes time derivatives.

2.1. *The governing equations*

We assume now that the motion of the rowers in the relative reference frame is assigned, namely $\mathbf{g}^{v^i} = \mathbf{g}^{v^i}(t) = (g_x^{v^i}(t), g_z^{v^i}(t))$ is the motion law for the baricenter of the i-th rower. It can be recast in the absolute reference frame by means of transformation (2). We can finally write the equations for the motion of a dynamical system composed by scull, oars, and oarsmen, as

$$\begin{aligned} M\ddot{\mathbf{G}}^c + \left(\mathcal{O}(\phi)\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\right)\ddot{\phi} = \\ -2\dot{\phi}\mathcal{O}(\phi)\sum_{i=1}^n M^{v^i}\dot{\mathbf{g}}^{v^i} - \dot{\phi}^2\mathcal{R}^T(\phi)\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i} - \mathcal{R}^T(\phi)\sum_{i=1}^n M^{v^i}\ddot{\mathbf{g}}^{v^i} + \\ + \mathcal{R}^T(\phi)\sum_{i=1}^n \mathbf{f}^{r^i}(t) + M\mathbf{g} + \mathbf{F}^a, \end{aligned} \tag{5a}$$

$$\begin{aligned} \left(\mathcal{R}^T(\phi)\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\right) \times \ddot{\mathbf{G}}^c + \left(I_y^c + \sum_{i=1}^n M^{v^i}\|\mathbf{g}^{v^i}\|^2\right)\ddot{\phi} = \\ -2\dot{\phi}\sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i}\cdot\dot{\mathbf{g}} - \sum_{i=1}^n M^{v^i}\mathbf{g}^{v^i} \times \ddot{\mathbf{g}} + \sum_{i=1}^n M^{v^i}(G_X^{v^i} - G_X^c)g + M^a. \end{aligned} \tag{5b}$$

Here, g is the module of the gravity acceleration ($9.81\ m/s^2$), M^{v^i} is the mass of the i-th rower, I_y^c is the moment of inertia of the boat around the y axis, while the matrix $\mathcal{O}(\phi)$ is defined as

$$\mathcal{O}(\phi) = \frac{d}{d\phi}\mathcal{R}^T(\phi) = \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & 0 & 0 \\ -\cos\phi & 0 & -\sin\phi \end{bmatrix}. \tag{6}$$

Fig. 1. The elliptic rower baricenter path in the XZ plane

The equations describing the rower path are

$$
\begin{aligned}
\tilde{g}^v_x(t) &= \tilde{x}_0 + a_x \cos(\theta(t)) \cos(\sigma) - a_z \sin(\theta(t)) \sin(\sigma) \\
\tilde{g}^v_x(t) &= \tilde{z}_0 + a_x \cos(\theta(t)) \sin(\sigma) + a_z \sin(\theta(t)) \cos(\sigma)
\end{aligned} \tag{1}
$$

Here $\tilde{x}_0 = (L^c + L^g)/2$, $\tilde{z}_0 = 4L^g$ and $a_x = (L^c + L^g)/2$, $a_z = L^g$. L^c is the excursion of the rowers seat, L^g is the distance between the rowers seat and their baricenter. The motion law, velocity and accelerations are readily computed from this expression.

Alternatively, we may use a trajectory inferred from telemetry measurements. Work is ongoing in this direction.

Fig. 2. The oarlock force during a period

3.3. *Forces due to the hull interaction with water*

The hydrostatic and hydrodynamic forces and moments are decomposed in the following way

$$\begin{aligned} \mathbf{F}^a &= S^a \mathbf{e}_Z - R^a \mathbf{e}_X + \mathbf{D}^a, \\ M^a &= M^a_S + M^a_D. \end{aligned} \tag{6}$$

Here S^a and M^a_S are the hydrostatic lift and moment respectively, and depend on the instantaneous position of the hull. The drag due to the primary motion R^a is computed by means of the empirical formula

$$R^a = \frac{1}{2}\rho S_{ref} C_{dX} (\dot{G}^c_X)^2, \tag{7}$$

being S_{ref} a reference surface (again depending on the instantaneous position of the boat), and C_{dX} a drag coefficient. The latter is computed for each boat by performing a Navier–Stokes simulation of the stationary motion.

Finally, the forces and moments due to the secondary motions of the boat, namely $\mathbf{D}^a$ and M^a_D, are computed by solving the following elliptic partial differential problems (see[5]) for the complex velocity potential ψ_α

$$\begin{cases} \Delta\psi_\alpha = 0 & \text{on } \Omega \\ \dfrac{\partial\psi_\alpha}{\partial z} - \dfrac{\omega^2}{g}\psi_\alpha = 0 & \text{on } \Gamma_{fs} \\ \dfrac{\partial\psi_\alpha}{\partial n} - i\dfrac{\omega^2}{g}\psi_\alpha = 0 & \text{on } \Gamma_\infty \\ \dfrac{\partial\psi_\alpha}{\partial n} = 0 & \text{on } \Gamma_b \\ \dfrac{\partial\psi_\alpha}{\partial n} = N_\alpha & \text{on } \Gamma_c \end{cases} \qquad \alpha = 1,2,3. \tag{8}$$

By a physical point of view, we solve three problems where a periodic motion (of frequency $f = \omega/2\pi$) is imposed to the boat surface, in the direction of each of the three degrees of freedom considered. The non-homogeneous Neumann conditions applied on the boat surface for each problem are therefore the components of the generalized normal vector $N = [n_x, n_z, zn_x - xn_z]$.

The forces due to secondary motions are finally computed by integrating the pressure obtained un the boat surface. It turns out that these forces present a component proportional to the acceleration vector $\ddot{\mathbf{u}}$ — giving rise to an added mass matrix $\mathcal{M}$— and a component proportional to the velocity vector $\dot{\mathbf{u}}$ — leading to a damping matrix $\mathcal{S}$. As for the angular velocity ω, we have taken it correspondingly to the principal frequency of the rowers motion.

4. The numerical solution

Introducing all these quantities in equations (5) we get a system of the form

$$A(t, \mathbf{y}(t))\frac{d\mathbf{y}}{dt}(t) = \mathbf{B}(t, \mathbf{y}(t)), \quad t > 0 \tag{1}$$

Fig. 1. Solution for a four crew boat

Other solutions are shown in Fig. 2 for two different single rower boats. One athlete (light line) weights 106 kg, the other (dark line) 85 kg, while they both push with $F_x^{max} = 1200\ N$. Here the average speeds are again compatible with real athletes ones. Moreover, we observe how the heavy rower's boat lowers more into the water, with respect to the light rower's one. This determines an increase of the wet surface, and therefore of the total drag, leading to a lower speed.

Fig. 2. Comparison between singles pushed by heavy (light line) and light (dark line) rowers

Capturing this physical mechanism, our model proves to be able to compute, for example, how much harder the heavier rower should push in order to compensate his weight disadvantage, and the additional energy needed to do it. This code therefore, can not only help boat designers, but also trainers and athletes in their strategies.

5. Conclusions

The algorithm presented here has proved to be robust and to produce physically correct results for any crew configuration tested. It is currently used to assess the effects of different geometries, rowing style and rower position on the boat. Still, improvements are necessary in several areas. In particular, the estimation of the oar forces and the inertia forces due to the rowers movement can benefit by analyzing experimental data coming from accelerometers and dynamometers placed on rowing boats, as well as telemetry devices.

Besides, the interfacing of the dynamical system for the boat motion with a different fluid-dynamic model based on the solution of Navier–Stokes equations with free surface, is ongoing. It will help to further validate the model.

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