

RISK MINIMIZING HEDGING IN A PARTIALLY OBSERVED HIGH FREQUENCY DATA MODEL: A FILTERING APPROACH

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Abstract

Risk-minimizing hedging strategies for contingent claims are studied in a general model for intraday stock price movements in the case of partial information. The dynamics of the risky asset price is described through a marked point process Y , whose local characteristics depend on some unobservable hidden state variable X , described by a jump-diffusion process. In the model presented the processes Y and X may have common jump times, which means that the trading activity may affect the law of X and could be also related to the presence of catastrophic events. The hedger is restricted to observing past asset prices. Thus we are in presence not only of an incomplete market situation but also of partial information. Assuming that the price of the risky asset is modeled directly under a martingale measure, the computation of the risk-minimizing hedging strategy under this partial information leads to a filtering problem with marked point process observations. The conditional law of X given the past asset prices (the filter) is characterized as the unique weak solution of the Kushner-Stratonovich equation. An explicit representation of the filter is obtained by the Feynman-Kac formula using a linearization method. This representation allows us to provide a recursive algorithm for the filter computation.

Keywords: High-frequency data, hedging under restricted information, risk minimizing hedging strategies, marked point processes, jump-diffusions, filtering.

Real asset prices, on a very small time scale, such as in intraday trading, are piecewise constant and jump only at discrete points in times in reaction to trades or to significant new information. Hence in a model for high-frequency data it is sensible to suppose that the price of assets can be described through marked point processes

In many papers, (see, for instance, [Fr] [FR1], [FR2], [MC], [RS] and [RZ]) the asset price process is modeled as a double stochastic Poisson process with marks. In some of them, the local characteristics of this process depend on an unobservable state variable, which may describe the intraday market activity, the activity of other markets, macroeconomics factors or microstructure rules that drive the market. In particular, in [Fr] the jump-intensity is assumed to depend on an exogenous state variable that is not directly observable for the agents. In [FR1] both the jump-intensity and the jump-size distribution depend on an unobservable state variable which is closely related to asset price volatility.

In this paper we consider, in the same framework of [Fr], [FR1] a more general model. The behaviour of the asset prices is described via a general marked point process Y , whose local characteristics, in particular the jump-intensity, depend on an unobservable state variable X , which is modeled by a Markov jump-diffusion process. Moreover, the dynamics of Y and X may be strongly dependent, in particular the two processes may have common jump times. Hence our model could take into account also the possibility of catastrophic events. This kind of events, in

fact, influence both the asset prices and the hidden state variable which drives their dynamics. Furthermore, the jump intensities both of Y and X are allowed to be unbounded.

In this note we are concerned with the hedging of contingent claims in which agents have access only to the information contained in past asset prices. When the given financial market is complete, every claim can be replicated by a self-financing dynamic portfolio strategy which only makes use of the existing assets. In this case, one can reduce to zero the risk of the claim by a suitable strategy. On the other hand, markets modeled by marked point processes, where infinite number of marks are allowed, are incomplete. Then one has to choose some approach to hedging derivatives. Since one cannot ask simultaneously for a perfect replication of a given claim by a portfolio strategy and the self-financing property of this strategy, one has to relax one of these conditions. There are two types of quadratic approaches to hedge a given square-integrable contingent claim: local risk minimization and mean-variance hedging. The mean-variance hedging keeps the self-financing condition but relaxes the replicability. On the other hand, the local risk minimization, which is the approach chosen in this paper, keeps the replicability and relaxes the self-financing condition. In [FSO] the authors dealt with the study of hedging of contingent claims under market incompleteness by introducing the criterion of risk minimization in the case when the price process is a martingale under the real world probability measure. In [S1] the more general semimartingale case has been considered, while, in [S2] the restricted information situation for the martingale case has been studied.

In this paper we study hedging of derivatives in incomplete market when just the information contained in past asset prices is available. As [Fr] and [FR1] we consider the case where the price of the risky asset is a local martingale under the real world probability measure. This assumption allows us to compute, by the Kunita-Watanabe decomposition, the hedge strategy assuming full knowledge of the latent state process X and then, by using a projection result ([S2]), to compute a risk-minimizing strategy for an agent who is restricted to observing the asset price process. This leads to the computation of conditional expectations of functions of X at time t given the sigma-algebra generated by Y until time t , which provides, at time t , all the available information. Thus, a filtering problem with marked point process observations arises. This problem is exhaustively studied in [CG] when Y is a discrete valued process. In this paper we extend the results proved in [CG] to the more general case in which Y is a real-valued process. More precisely, we derive the Kushner-Stratonovich equation and we give an explicit expression for the filter via the Feynman-Kac formula, where just ordinary expectations appear. According to the observed jump times of Y , this procedure provides a recursive algorithm for the computation of the filter, whenever the solutions of a suitable stochastic differential equation can be simulated. These results provide a complete solution to the hedging problem.

Based on the following papers:

- C.Ceci-A.Gerardi: "A model for high frequency data under partial information: a filtering approach", Preprint (2005), Univ. dell'Aquila, <http://www.diel.univaq.it/intranet/index.php> in corso di pubblicazione su International Journal of Theoretical and Applied Finance.
- C.Ceci: "Risk minimizing hedging for a partially observed high frequency data model", Stochastics: An International Journal of Probability and Stochastic Processes 78 (1) (2006) 13-31.

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