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Catastrophic Events, Claims and Risks

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Recently, quite a lot of studies have been devoted to particular events usually denominated natural or man–made "catastrophes".

As it results from a sample of expressions and definitions extracted from the existing literature, there are different ways of expressing the same ideas regarding such topics that, at the moment, are not entirely inserted into a logical structure.

The scope of the present paper is to provide such a logical structure and, accordingly to it, to suggest some technical definitions that may be utilized to get a standardization, mostly in the actuarial setting, of the expressions used. Such definitions, particularly that one concerning the catastrophic risk, may be useful as a tool for evaluating the insured risks' portfolio of a property–casualty insurance company, especially in order to outline an optimal reinsurance strategy.

For this purpose we start from the analysis of the word "catastrophe". It is clear that "catastrophe" can be referred both to an "extreme event" and to the "claim" caused by the event itself, which may be intended both as a random variable (before the realization of the event) and a deterministic "loss" (after the event has occurred). Moreover, "catastrophe" is generally referred to an event which usually involves "a lot of" people rather than a single one.

When it is intended as "extreme event", the meaning associated to such a word is provided in an absolute sense and, hence, it is defined a priori. When the "catastrophe" is intended as a "claim" and, therefore, to whom it may concern, the meaning associated to such a word is relative, thus it can be defined only a posteriori. Obviously, for an exhaustive approach to the matter we should avoid to privilege one sense rather than the other one. The word "catastrophe" is not exhaustive in this context and it must be substituted by expressions lacking of ambiguity: "extreme event" and "claim" can be both utilized in this sense. Since in the actuarial setting the main concept is the "claim" suffered by a subject, we think more suitable a relative approach to the matter and, consequently, we adopt the adjective "catastrophic".

In the light of the existing literature and mainly for the end of this paper, the subjects to be considered can be assembled into the following four categories: (s_1) the single individual, (s_2) the society, intended, both as "community" and as "state", (s_3) a generic "property–casualty insurance company" (briefly, i.c.) or "reinsurance company" (r.c.),

(s_4) the set of firms (mainly, the i.c.'s and the r.c.'s) directly involved in the insurance management, which we briefly refer to as "insurance market" (i.m.).

In regard to the link among the basic concepts concerning the matter, the "risk" and the"claim" are, distinctly in time, the "event's outcomes" for the "subject".

This last consideration also yields the necessity to distinguish a "before" and an "after" scenario, that is, to introduce a time division between what is "potential" and what is actually so. Having denoted by the random variable (r.v.) $\tau > 0$ the epoch in which the event may happen, when we consider either s_1 or s_2 the before scenario is the random interval $[0, \tau]$ while, when we concentrate on the i.c., as well as on i.m., the before scenario is any fixed deterministic interval corresponding to the period chosen for the evaluation of the total claim amount related to its insured risks' portfolio.

In the before scenario a *potentially catastrophic claim* is defined through some criteria, established according to the subject s in question; an event is potentially catastrophic for s if and only if the (random) claim related to such an event's realization is potentially catastrophic for it; a *catastrophic risk* (c.r.) for s is defined in terms of the *potentially catastrophic event*, its probability of occurrence and the potentially catastrophic claim for s. In particular, for s_1 and s_2 the risk is estimated in every future time while for s_3 and s_4 the evaluation of the risk is based on the evaluation of the total claim amount of a portfolio of insured risks made in a fixed time horizon (during which the portfolio is supposed to be invariant) assumed, for simplicity, equal to one year. In the following we describe the before scenario concentrating on the subject s_3.

Regarding the claim, we distinguish two cases: (i) the potentially catastrophic claim $S_{3,1} = S_Y$, giving the r.v. total claim amount of the underlying portfolio Y in the predefined interval; (ii) the potentially catastrophic claim $S_{3,2} = S_{\widetilde{Y}}$, which expresses the total claim amount of the portfolio $\widetilde{Y} = \{Y; X\}$, where X is a "unity of risk" (i.e. a random vector made of $n > 1$ dependent components, a single risk when $n = 1$, each one related to the same event) added to an invariant portfolio Y whose management is *authorized* (in a sense that will be defined below).

More formally, having denoted by $[0, M(S_{3,j})]$ the support of the r.v. $S_{3,j}$, $j = 1, 2$, let $\widehat{M}(S_{3,j})$ be the value corresponding to the greatest probability per year that s_3 approximates to zero, that is,

$$\overline{F}_{S_{3,j}}(\widehat{M}(S_{3,j})) \approx 0, \qquad \widehat{M}(S_{3,j}) \in (\bar{l}_{3,j}, M(S_{3,j})), \qquad \bar{l}_{3,j} > 0. \tag{0.1}$$

Under condition (0.1) we say that $S_{3,j}$ is a potentially catastrophic claim when it fulfils the following condition:

$$\frac{\widehat{M}(S_{3,j})}{\bar{l}_{3,j}} > K, \qquad \text{for a given } K > 1. \tag{0.2}$$

Particularly, for $S_{3,1} = S_Y$,

$$\bar{l}_{3,1} = \max_{m_\alpha \le \alpha \le M_\alpha, m_\eta \le \eta \le M_\eta} CS(Y), \tag{0.3}$$

while for $S_{3,2} = S_{\widetilde{Y}}$,

$$\bar{l}_{3,2} = \max_{\widetilde{m}_\alpha \le \widetilde{\alpha} \le M_\alpha, \widetilde{m}_\eta \le \widetilde{\eta} \le \widetilde{M}_\eta} CS(\widetilde{Y}). \tag{0.4}$$

In these formulas CS denotes the *capital structure* at the i.c.'s disposal whose maximum value is represented by $\bar{l}_{3,j}$ ($\bar{l}_{3,1} \neq \bar{l}_{3,2}$ because of the different range for the pairs (α, η) and $(\widetilde{\alpha}, \widetilde{\eta})$ which define the capital structures $CS(Y) = CS(\alpha, \eta; Y)$ and $CS(\widetilde{Y}) = CS(\widetilde{\alpha}, \widetilde{\eta}; \widetilde{Y})$, respectively). We say that the management of the portfolio is *authorized* if $CS \geq CS_{min}$ where CS_{min} represents the minimum capital structure necessary to run the portfolio according to a set of criteria established by the i.c. itself and some constraints and rules imposed on the i.c. by both the i.m. and an external regulatory authority's [1]. Particularly, CS_{min} is fixed taking the values of $\widehat{M}(S_{3,j})$ and K into account.

From the definitions we propose there results that a potentially catastrophic claim for s_3 corresponds to a claim that may "potentially" exceed K times the maximum capital structure of the i.c. and it is easy to see that this yields a not authorized portfolio's management. Therefore, the related risk, Y in the case (i), X in the case (ii), cannot be completely retained and a part of it must be necessarily ceded in reinsurance.

Having in mind the definitions of potentially catastrophic claim, we say that the sequence of all the events related to the portfolio Y (briefly, the "event related to Y") is potentially catastrophic if $S_{3,1}$ satisfies conditions (0.2),(0.3) and the event related to $\widetilde{Y} = \{Y; X\}$ is potentially catastrophic if $S_{3,2}$ satisfies conditions (0.2),(0.4).

For the definition of catastrophic risk we start from the general definition of risk provided by the ISO/IEC guide: "the risk is a combination of the probability of an event and its consequence" [2]. We think that such a definition has been argued only for a single risk, that is when a single event may constitute a risk for a single "individual".

We then suggest to define the catastrophic risk as the following set:

$$\text{catastrophic risk} = \left\{ \begin{array}{l} \text{potentially catastrophic event, probability,} \\ \text{potentially catastrophic claim.} \end{array} \right\}$$

As the claim and the event are potentially catastrophic according to s_i, it must always be considered the pair $\{$subject, catastrophic risk$\}$.

More formally, we say that a subject s_i, $i = 1, \ldots, 4$, is exposed to a catastrophic risk if a potentially catastrophic event can occur, that is, if there exists an event whose realization is related to a potentially catastrophic claim for s_i.

Particularly, for s_3, we introduce two different classes of catastrophic risk: the c.r. represented by the portfolio Y, related to a potentially catastrophic claim $S_{3,1}$ satisfying conditions (0.2),(0.3), and the c.r. represented by the unity of risk X, related to a potentially catastrophic claim $S_{3,2}$ satisfying conditions (0.2),(0.4).

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