

Banks' optimal rating systems and procyclicality

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Abstract

The introduction of Basel II has rised concerns about the possible impact of risk-sensitive capital requirement on the business cycle. Several approaches have been proposed to deal with the procyclicality issue: a general equilibrium model is an appropriate framework for a comprehensive analysis of different proposals since it allows to account for banks endogenous strategies in relation to the other agents' behaviour. The set up of a model to evaluate different rating systems in relation to the procyclicality issue is presented.

1 Introduction

The introduction of Basel II has rised concerns about the possible impact of risk-sensitive capital requirement on real economy: since it is widely recognised that credit risk factors are affected by economic conditions (see e.g. [1] and [9]), risk sensitive capital requirement would fluctuate over the business cycle, possibly causing an amplification of the same through the lending channel. In particular, the concerns focus on the possible exacerbation of recessions due to credit shortage.

[8] define three possible ways to deal with procyclicality under the IRB Basel framework. The first two ways consist in smoothing the input to the capital function (mainly adopting a *through the cycle (ttc)* rating philosophy) or flattening the capital function itself (by reducing the sensitivity to the PDs). These two ways have partly been followed in moving from the first version of the Basel document to [2]. However both these ways lead to a loss in risk sensitivity, since they play on the trade-off between risk-sensitivity and procyclicality.[8] advocate the third way, which consists in smoothing the output of the capital function for regulatory purpose and allows to avoid losses in terms of transparency. In

[10] an alternative approach is proposed where a business cycle forecast is included into the PDs estimation: this is consistent with the views expressed in [3]: the risk is built up during expansions and the high default rates observed during recessions are just a materialization of that risk. According to this view, a risk measure should be high in expansion before a recession and low at the bottom of the cycle in anticipation of an expansion: a capital requirement based on such a risk measure would decline at the bottom of the cycle helping the economy out of the recession.

In order to evaluate different proposal to deal with the procyclical-ity issue, in line with [4] we propose to use a general equilibrium (GE) model. [8] compare different rating philosophies by considering banks with different exogenous investment strategies: however, in this way the feedback impact of capital requirement on banks lending behaviour and hence on the economy is not accounted for. A general equilibrium model is an appropriate framework for a comprehensive analysis of the procyclical-ity issue since it allows to account for banks endogenous strategies to be made consistent with the other agents' behaviour.

[4] analyse the procyclical-ity issue by comparing different rating systems from the banks' profitability point of view: within a 2-periods, 2-states GE model with one bank, one corporate and one household, they find that banks would prefer *point in time (pit)* rather than *ttc* rating system, with dangerous consequences in terms of procyclical-ity. However they do not account for the heterogeneity of agents, while we think this is an important feature in this context. The model used in [4] originally comes from the general theoretical model presented in [11] and [5]. [7] presents a smaller version of the original model which allows to obtain numerical solution while [6] simplify the non-banking agents problems to reduced-form equations in order to perform a calibration against real UK banking data. Building on these models we propose a framework to analyse the procyclical-ity issue.

In the following Section we present the agents' optimization problems and the aim of the model. Section 3 concludes and define the line of research we are going to follow.

2 The model

The model we propose is a general equilibrium model of an exchange economy with money and bank, following the lines defined in [11]. Three sectors are considered: banking sector, corporate sector and households. While banks maximize a function of their profit, corporate and households aim at smoothing the intertemporal consumption. The model is characterized by heterogeneity of the agents, limited participation of

banks towards corporate and household and endogenous default. The structure of the model is synthetised in the following points:

- 1) Two periods $t=0,1$
- 2) Two states of the world in $t=1$: E (expansion),R (recession)
- 3) Two banks: bank γ is a net borrower on the interbank market; bank δ is a net lender on interbank market; both banks maximize expected utility of profits in $t=1$ subject to penalties on default and capital requirement constraints;
- 4) Two corporates α, β : each corporate borrows from a single bank and maximize expected utility of consumption minus a default penalty;
- 5) Two households φ, θ : each household deposits in a single bank and maximize expected utility of consumption;
- 6) Central bank/regulator exogenously define: money supply (M), minimum capital requirement in terms of capital ratio (K), default penalties (λ).

The optimization problem for the single agents are presented in the following.

2.1 Banks' optimization problem

There are two simultaneous optimization problems for the two heterogeneous banks. Both banks maximize the expected value of a quadratic function of profit reduced by a default penalty and are subject to a capital constraint in the Basel II style.

The bank γ is a net borrower on the interbank market and its optimization problem is defined in the following. The bank maximize the expected utility of profits in $t=1$ less a penalty in case of she defaults on deposits or interbank debts.

$$\max_{m_\alpha^\gamma, A^\gamma, \mu^\gamma, \mu_d^\gamma, v_s^\gamma} \sum_s p_s [f(\pi_s^\gamma) - \lambda^\gamma (\mu^\gamma + \mu_d^\gamma) (1 - v_s^\gamma)] \quad (1)$$

The bank is subject to the constraints 2 and 3 in $t=0$ and $t=1$ respectively:

$$m_\alpha^\gamma + A^\gamma = \frac{\mu^\gamma}{1 + \rho} + \frac{\mu_d^\gamma}{1 + r_d^\gamma} + c_0^\gamma \quad (2)$$

$$v_s^\gamma (\mu^\gamma + \mu_d^\gamma) \leq v_s^\alpha (1 + r_m^\gamma) m_\alpha^\gamma + (1 + r_A) A^\gamma \quad (3)$$

In $t=0$ the investments are limited to the money coming from deposits and interbank borrowings plus capital, while in $t=1$ revenues from investments must cover the repayments to all creditors. Moreover the bank is subject to the capital constraint 4 in $t=0$

$$k^\gamma \geq K \quad (4)$$

where:

- p_s = probability of state s in $t=1$;
- π_s^γ = profit of bank γ in state s ($t=1$);
- m_α^γ = loan extension to corporate α ;
- A^γ = investment in default free assets;
- μ^γ = money borrowed on the interbank market;
- μ_d^γ = money from deposits;
- v_s^γ = repayment rate of bank γ to all creditors;
- v_s^α = repayment rate of corporate α to all creditors;
- c_0^γ = initial capital of bank γ ;
- λ = default penalty;
- r_d^γ = deposit interest rate;
- r_A = interest rates on default-free asset A ;
- ρ = interbank interest rate;
- k^γ = bank γ ' capital ratio.

The profit is defined in 5 and the capital ratio in 6

$$\pi_s^\gamma = v_s^\alpha (1 + r_m^\gamma) m_\alpha^\gamma + (1 + r_A) A^\gamma - v_s^\gamma (\mu^\gamma + \mu_d^\gamma) - c_0^\gamma \quad (5)$$

$$k^\gamma = \frac{c_0^\gamma}{w (1 + r_m^\gamma) m_\alpha^\gamma + \bar{w} (1 + r_A) A^\gamma} \quad (6)$$

The terms w and \bar{w} are the Basel II risk weights: specifically, as explained in Section 2.4, the weight w on risky assets can be defined as to account for different rating philosophies. Credit risk for banks arises from corporates possible default

The bank δ is a net lender on the interbank market and this makes its optimization problem slightly different from the previous one in the position one. Moreover, while corporate α is the counterpart for bank γ , corporate β is the counterpart for bank δ .

$$\max_{m_\beta^\delta, A^\delta, \mu^\delta, \mu_d^\delta, v_s^\delta} \sum_s p_s [f(\pi_s^\delta) - \lambda^\delta (\mu^\delta + \mu_d^\delta) (1 - v_s^\delta)] \quad (7)$$

Subject to the constraints 8 and 9 in $t=0$ and $t=1$ respectively:

$$m_\beta^\delta + A^\delta + d^\delta = \frac{\mu_d^\delta}{1 + r_d^\delta} + c_0^\delta \quad (8)$$

$$v_s^\delta \mu_d^\delta \leq v_s^\delta (1 + r_m^\delta) m_\beta^\delta + (1 + r_A) A^\delta + v_s^\delta d^\delta (1 + \rho) \quad (9)$$

and subject to the capital constraint 10 in $t=0$

$$k^\delta \geq K \quad (10)$$

where:

- π_s^δ = profit of bank δ in state s ($t=1$);
- m_β^δ = loan extension to corporate β ;
- A^δ = investment in default free assets;
- μ_d^δ = money from deposits;
- v_s^δ = repayment rate to all creditors;
- r_d^δ = deposit interest rate;
- c_0^δ = initial capital of bank δ ;
- k^δ = bank δ ' capital ratio.

The profit is defined in 5 and the capital ratio in 6

$$\pi_s^\delta = v_s^\beta (1 + r_m^\delta) m_\alpha^\delta + (1 + r_A) A^\delta + v_s^\gamma d^\delta (1 + \rho) - v_s^\delta \mu_d^\delta - c_0^\delta \quad (11)$$

$$k^\gamma = \frac{c_0^\gamma}{w (1 + r_m^\gamma) m_\alpha^\gamma + \bar{w} (1 + r_A) A^\gamma + \tilde{w} (1 + \rho) d^\gamma} \quad (12)$$

2.2 Corporates' optimization problem

The economy is one without production and the stochastic endowments replace production output. The corporates are poor agents in $t=0$ and they receive stochastic commodity and money endowments in $t=1$; therefore they borrow in $t=0$ in order to smooth intertemporal consumption. The two corporates are different in endowments but their optimization problems can be synthetised in a single problem.

$$\max_{b_0^h, q_s^h, \mu^h, v_s^h} f(x_0^h) + \sum_s p_s [f(x_s^h) - \lambda^h \mu_b^h (1 - v_s^h)] \quad (13)$$

Subject to the constraints 14 and 15 in $t=0$ and $t=1$ respectively:

$$b_0^h \leq \frac{\mu^h}{1 + r_m^\delta} \quad (14)$$

$$v_s^h \mu^h \leq \left(\frac{\mu^h}{1 + r_m^\delta} - b_0^h \right) + g_s q_s^h + m_s^h \quad (15)$$

where:

- $h = \alpha, \beta$
- $b = \gamma, \delta$
- x^h = real consumption (commodity expenditures/commodity prices):
- $x_0 = b_0^h/g_0; x_s^h = e_s^h - q_s^h;$
- e_s^h = commodity endowment in $t=1$;
- m_s^h = monetary endowment in $t=1$;

b_0^h = expenditures for commodities;
 q_s^h = quantity of commodity to sell;
 μ^h = borrowings from the assigned bank;
 v_s^h = repayment rate;

2.3 Households' optimization problem

The households are rich agents in period $t=0$ and deposit in the banks in order to produce a consumption smoothing.

$$\max_{d_b^l} f(x_0^l) + \sum_s p_s f(x_s^l) \quad (16)$$

Subject to the constraints 17 and 18 in $t=0$ and $t=1$ respectively:

$$d_b^l \leq m_0^l; \quad q_b^l \leq e_0^l \quad (17)$$

$$b_s^l \leq (m_0^l - d_b^l) + g_0^l q_0^l + v_s^b d^l (1 + r_d^b) \quad (18)$$

where:

$l = \phi, \theta$

$b = \gamma, \delta$

x^h = real consumption (commodity expenditures/commodity prices):

$x_0^l = e_0^l - q_0^l; \quad x_s^l = b_s^l/g_s;$

e_0^l = commodity endowment in $t=0$;

m_0^l = monetary endowment in $t=0$;

d_b^l = deposits;

b_s^l = expenditures for commodities.

2.4 Model at work

The model can now be solved accounting for the market clearing conditions. The clearing conditions on commodity market, loans markets, deposit markets and interbank market are shown in respectively.

$$g_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi + q_0^\theta} \quad g_s = \frac{b_s^\phi + b_s^\theta}{q_s^\alpha + q_s^\beta} \quad (19)$$

$$1 + r_m^b = \frac{\mu^h}{m^b} \quad (20)$$

$$1 + r_d^b = \frac{\mu_d^b}{d^l} \quad (21)$$

$$1 + \rho = \frac{\mu^\gamma}{d^\delta + M} \quad (22)$$

Once the model is solved we want to use it to perform analyses related to the procyclicality issue. First of all, in order to account for different rating philosophies the risk weight on risky assets in the capital ratio formula can be modelled as a function of the expected repayment rates:

$$w = F \left(\sum_s p_s v_s^h \right) \quad (23)$$

The functional form of F can be used to model *ttc* or *pit* rating. The business cycle states are represented by means of the endowments and the modelisation of the state probabilities drives the relation between capital requirements and the real economy. Within this framework we can compare different rating systems from the banks' point of view. Moreover, given a rating system, we can analyse the business cycle feedback effects through the total credit extension: this allows a more general welfare analysis.

2.5 Conclusion and further research

The procyclicality issue requires a comprehensive framework to be adequately analysed. In the present article we set up a general equilibrium model with appropriate features to analyse the relation between capital requirements, credit extension and the business cycle. The next step in our research is to solve the model and use it to analyse different rating systems both from the banks' profitability and from a general welfare point of view.

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