

Active Infrared Thermography in Nondestructive Evaluation (2): Detection of Hidden Damages from Real Data

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The goal of NonDestructive Evaluation by means of InfraRed Thermography (NDE via IRT) is the detection of subsurface anomalies in a conductor Ω . In this paper we report about a method (based on the analysis in Bison et al.(2005) where you can find a rich bibliography) for detecting damages from real data when Ω is a plate. Experiments are taken at the CNR-ITC laboratory in Padova.

We model the specimen Ω by means of the open parallelepiped

$$\Omega = (0, 1) \times (0, 1) \times (0, a)$$

where the *thickness* a is assumed to be much lower than 1. Here, we are able to control heat fluxes and to measure temperature responses at the bottom face $(x, y, 0)$ while the top one (x, y, a) turns out to be inaccessible. Since Ω has been assumed to be very thin ($a \ll 1$), defects on the inaccessible face can be regarded as subsurface anomalies from the point of view of subjects operating at the face $z = 0$. Assume that Ω is an isotropic and homogeneous heat conductor. Its *conductivity tensor* is $\kappa_0 I$: I is the identity matrix 3×3 and κ_0 is constant in space. The *volumetric heat capacity* $C_0 > 0$ is also constant. Moreover, during the test, the temperature field varies in a range small enough to make negligible any dependence of κ_0 and C_0 from the temperature itself.

Let $u^0(x, y, z, t)$ be the (background) temperature in Ω for $t \in (0, T)$. The function u^0 solves the heat equation

$$(1.1) \quad u_t^0 = \frac{1}{C_0} \nabla \cdot \kappa_0 \nabla u^0 \quad \text{in } \Omega \times (0, T),$$

with initial data $u^0(x, y, z, 0) = g^0(x, y, z)$ in Ω and boundary conditions:

$$(1.1.a) \quad u_x^0(0, y, z, t) = u_x^0(1, y, z, t) = 0$$

$$(1.1.b) \quad u_y^0(x, 0, z, t) = u_y^0(x, 1, z, t) = 0$$

$$(1.1.c) \quad u_z^0(x, y, 0, t) - \gamma_{bot}[u^0(x, y, 0, t) - u_{bot} - \frac{1}{h_{bot}}\phi(x, y, t)] = 0$$

$$(1.1.d) \quad u_z^0(x, y, a, t) + \gamma_{top}[u^0(x, y, a, t) - u_{top}] = 0$$

$x, y \in (0, 1)$, $z \in (0, a)$, $t \in (0, T)$. Here γ_{bot} and γ_{top} are constant positive coefficients defined as

$$\gamma_{bot} = \frac{h_{bot}}{\kappa_0} \quad \gamma_{top} = \frac{h_{top}}{\kappa_0}$$

h_{bot} and h_{top} being the *heat exchange coefficients* between the specimen surfaces and the environment on bottom and top sides respectively. u_{bot} and u_{top} are external temperature values (see for example the classical book by Carslaw and Jaeger), while the function ϕ in (1.1.c) describes the heating of our specimen through the accessible side of Ω . Furthermore, let $f^0(x, y, t_k) = u^0(x, y, 0, t_k)$ be the (background) temperature maps collected at instants $0 = t_0 < t_1 < \dots < t_N = T$.

If we assume that the effects of external aggressiveness are modelled by some material loss from the inaccessible side, the damaged specimen is represented as $\Omega_{\epsilon\theta} = \{(x, y, z) : 0 < x, y < 1, 0 < z < a - \epsilon\theta(x, y)\}$ with $\epsilon \ll a$. Here θ is a smooth function such that $\theta(0, y) = \theta(1, y) = \theta(x, 0) = \theta(x, 1) = 0$ and $0 \leq \theta(x, y) \leq 1$. The support of θ (i.e., the open set where $\theta > 0$) is strictly included in $(0, 1) \times (0, 1)$. In particular, the constant ϵ plays the role of an a-priori bound for the extent of loss of matter which is described by the nonnegative function $\epsilon\theta$. It is reasonable to assume that corrosion (or any other damaging process) acts slowly in the time scale of our experiments. Hence, $\epsilon\theta$ is assumed to be time-independent in what follows. Temperature in $\Omega_{\epsilon\theta}$ still satisfies heat equation (1.1) but the boundary conditions on the top side of the domain are clearly modified.

As for the initial condition, we can assume that the value of u for $t = 0$ is not dependent on the perturbation $\epsilon\theta$. This assumption should require some more comment if we did not assume a periodic ϕ in what follows.

The temperature maps $f(x, y, t_k) = u(x, y, 0, t_k)$, collected at instants $0 = t_0 < t_1 < \dots < t_N = T$ are modified as an effect of corrosion. The discrepancies $f(x, y, t_k) - f^0(x, y, t_k)$ (*contrast* in temperature response) and the flux data ϕ will be used in the following to recover $\epsilon\theta$.

We introduce a new system of coordinates in order to retransform, by means of the map $\Phi : \Omega_{\epsilon\theta} \mapsto \Omega$, the damaged region $\Omega_{\epsilon\theta}$ in the original domain Ω : $\xi = \Phi_1(x, y, z) = x$, $\eta = \Phi_2(x, y, z) = y$, $\zeta = \Phi_3(x, y, z) = z + \epsilon\theta(x, y)\psi(z)$ where $\psi : [0, a] \rightarrow [0, 1]$ is any non decreasing, smooth function such that $\psi(0) = 0$ and $\psi(a - \epsilon) = 1$. Assume also that $\psi'(0) = 0$; clearly it is $\psi'(z) = 0$ for $z \in (a - \epsilon, a)$.

Apply the change of variable Φ to the heat equation $C_0 u_t = \nabla \cdot \kappa_0 \nabla u$ in $\Omega_{\epsilon\theta} \times (0, T)$. To be more precise, we define a new function $v = u \circ \Phi^{-1} : \Omega \rightarrow \mathbb{R}$.

It comes from standard calculations that $\nabla \cdot \nabla u = |J\Phi| \nabla \cdot \frac{(J\Phi)(J\Phi)^T}{|J\Phi|} \nabla v$ where $J\Phi$ is the jacobian matrix of Φ , the superscript T denotes transposition, and $|J\Phi|$ is the jacobian determinant.

Since $u_t(x, y, z, t) = v_t(\Phi_1(x, y, z), \Phi_2(x, y, z), \Phi_3(x, y, z), t)$, heat equation (1.1) becomes:

$$(1.2) \quad \frac{C_0}{|J\Phi|} v_t = \nabla \cdot \kappa_0 \frac{(J\Phi)(J\Phi)^T}{|J\Phi|} \nabla v$$

$\forall (\xi, \eta, \zeta, t) \in \Omega \times (0, T)$. The physical properties of the specimen are changed. We have that $C = \frac{C_0}{|J\Phi|} = C_0(1 + O(\epsilon))$ is the new heat capacity, while $\kappa = \kappa_0 \frac{(J\Phi)(J\Phi)^T}{|J\Phi|} = \kappa_0(1 + O(\epsilon))$ is the new conductivity.

Boundary conditions for v are essentially the same as for u^0 , due to the hypotheses on ψ . It comes that any solution of a boundary value problem for (1.2) is C^∞ in ϵ as consequence of regularity of κ and C thanks to the implicit function theorem and the well posedness of (1.2). Consider the Taylor expansions $\kappa = \kappa_0 + \epsilon \kappa_1 + O(\epsilon^2)$ and $C = C_0 + \epsilon C_1 + O(\epsilon^2)$ where

$$\kappa_1 = \kappa_0 \begin{pmatrix} -\theta\psi_\zeta & 0 & \theta_\xi\psi \\ 0 & -\theta\psi_\zeta & \theta_\eta\psi \\ \theta_\xi\psi & \theta_\eta\psi & \theta\psi_\zeta \end{pmatrix}$$

$$C_1 = -C_0\theta\psi_\zeta \text{ and } v = v_0 + \epsilon v_1 + O(\epsilon^2)$$

If we plug these expansions in (1.2) derive a hierarchy of relations, typical of perturbative methods. The relation of order zero is equation (1.1). Boundary conditions of order zero are just (1.1.a,b,c,d) so that v_0 is actually the background solution u^0 .

The first order in the previous expansion gives the parabolic PDE in $\Omega \times (0, T)$

$$(1.3) \quad C_0 v_{1t} = \kappa_0 \Delta v_1 + \nabla \cdot k_1 \nabla v_0 - C_1 v_{0t}$$

with boundary conditions

$$\begin{aligned} \epsilon v_1(\xi, \eta, \zeta, 0) &= g - g^0 \\ v_{1\zeta}(\xi, \eta, 0, t) - \gamma_{bot} v_1(\xi, \eta, 0, t) &= 0 \\ v_{1\eta}(\xi, 0, \zeta, t) = v_{1\eta}(\xi, 1, \zeta, t) = v_{1\xi}(0, \eta, \zeta, t) = v_{1\xi}(1, \eta, \zeta, t) &= 0 \\ \kappa_0 v_{1,\zeta} + h v_1(\xi, \eta, a, t) &= -(\theta_\xi v_{0,\xi} + \theta_\eta v_{0,\eta}). \end{aligned}$$

This boundary value problem improves the similar model studied in Bison et al. (2005). In that paper no effect on the top boundary condition of a non constant conductivity tensor in the new coordinates was considered.

We recall that, following Bison et al.(2005), the heating term ϕ is assumed periodic in time with period τ . It means that after a transition time, the temperature can be regarded as periodic with period τ . Fourier expansion of v_1 transform (1.4) and its boundary conditions in a family of elliptic equations.

We made a number of numerical experiment for the three dimensional case to show the performance of our procedure; we discretized the equations by means of finite differences using, for simplicity, the same spatial step ($h=0.01$) for all of the direction. Then

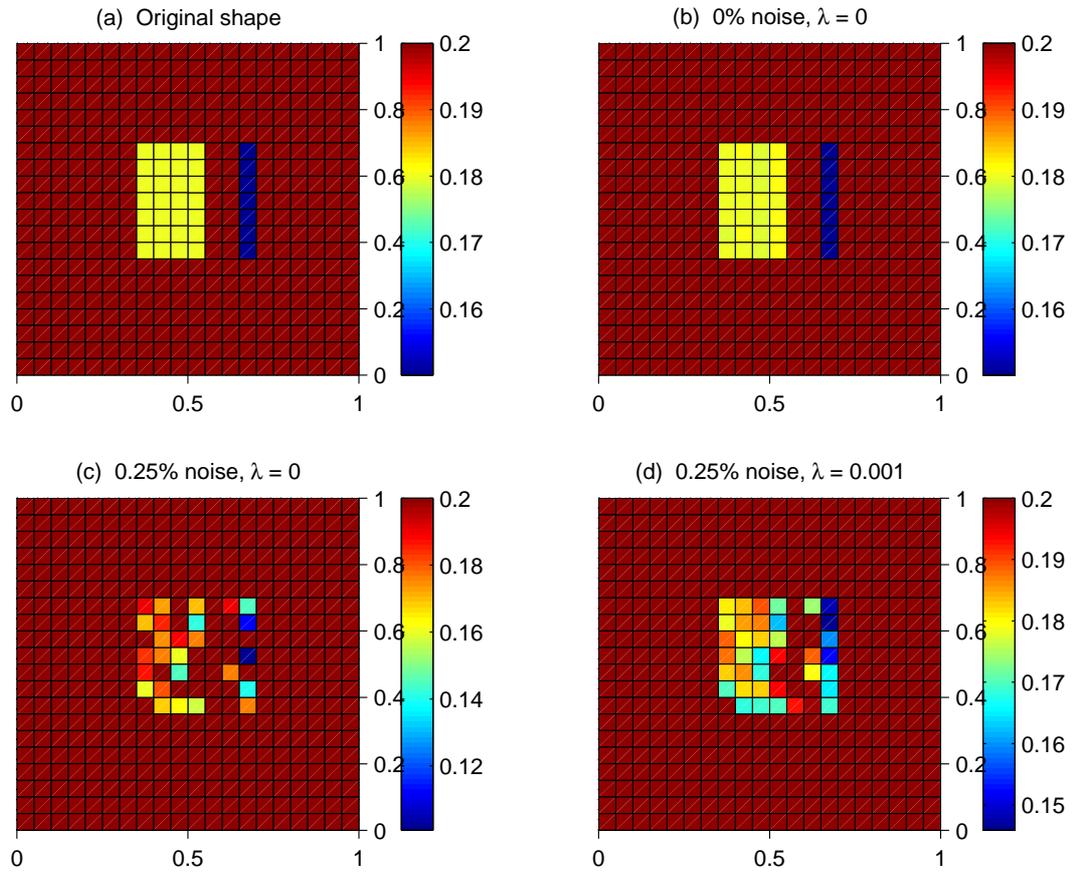


Figure 0.1: Reconstruction of $\epsilon\theta$: (a) original shape, (b) reconstruction from exact data, (c) reconstruction from noisy data (0.25%) without regularization and (d) with regularization.

the function $\epsilon\theta$ is approximated by means of two dimensional step function. A regularized (Tikhonov) output least square method lead to the reconstruction of damages from sintethic data shown in figure 1. Real data processing is in progress.

REFERENCES

1. P. Bison, D.Fasino and G. Inglese, in *Series on Advances in Mathematics for Applied Sciences Vol 69 - Proceedings of the 7th SIMAI Conference - Venice 2004* World Scientific: New Jersey, 2005, 143–154.