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Collision Avoidance Problem for an Ekranoplan

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Abstract

The risk of collision is one of the crucial factor for the applications of Ekranoplans in civil transportation. In fact, the extremely low flight altitude of these aircraft increases dramatically the chances of interference between their flight path and the multitude of obstacles populating the surrounding area. Examples of those potential obstacles are: ships, small boats, and stumbling blocks.

In this work we consider the optimal collision avoidance problem between a cruising Ekranoplan and a steady obstacle located on the ground. The following assumptions are employed: (i) initially the Ekranoplan is moving in a quasisteady levelled cruise trajectory; (ii) after the avoidance manoeuvre a recovery manoeuvre is executed by the aircraft to return to the initial cruise path; (iii) both the avoidance manoeuvre and the recovery manoeuvre lie on the same vertical plane identified by the initial cruise trajectory. The investigation of the collision avoidance performances on the longitudinal flight is encouraged by the relatively large quasi-flat turn radius of the Ekranoplans.

Keywords: `Collision avoidance, ekranoplan, optimal control, Chebyshev problems, Bolza problems, transformation techniques, multiple-subarc sequential gradient-restoration algorithm.`

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1. Introduction.

The risk of collision is one of the crucial aspects for the applications of Ekranoplans in civil transportation. In fact, the extremely low flight altitude of these aircrafts increases dramatically the chances of interference between their flight path and the multitude of obstacles populating the surrounding area. Examples of those potential obstacles are: ships, small boats, and stumbling blocks.

In this work we investigate the optimal collision avoidance problem between a cruising Ekranoplan and a steady obstacle located on the ground. The following assumptions are employed: (i) initially the Ekranoplan is moving along a quasisteady levelled cruise trajectory; (ii) after the avoidance manoeuvre a recovery manoeuvre is executed by the aircraft to return on the initial cruise path; (iii) both the avoidance manoeuvre and the recovery manoeuvre lie on the same vertical plane identified by the initial cruise trajectory. The investigation of the collision avoidance performances on the longitudinal flight is encouraged by the relatively large quasi-flat turn radius of the Ekranoplans.

The approach to the problem considered, is to maximize wrt the controls the timewise minimum distance between the aircraft and the obstacle. This yields to a maximin problem or Chebyshev problem of optimal control, which is not solvable in a direct way. Hence, a technique is performed to transform the Chebyshev problem into a Bolza problem (Ref. 1). Once reduced in this form the optimization problem can be solved numerically applying the multiple-subarc sequential gradient-restoration algorithm (Refs. 2-3).

The main target of this research is to determine the relationship between the optimal avoidance maneuver and the control to execute it. In turn, this relationship is basilar to the development of a guidance scheme capable to approximate the optimal trajectory in real time.

It is worth to notice that the peculiar aerodynamic characteristics of the Ekranoplans joined to their relatively weak manoeuvrability make this application of optimal control techniques particularly challenging. We believe that the results of the present work would lead the future investigations toward a suitable collision avoidance strategy.

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2. Problem Description.

The motion of the aircraft is described considering the following assumptions: (i) the flight path lies in a vertical plane; (ii) the aircraft is a particle of constant mass; (iii) the Earth is flat; (iv) the Earth-fixed coordinate system is inertial. Under these conditions, the equations of motion can be written as

$$x' = V \cos\gamma, \tag{1a}$$
$$h' = V \sin \gamma, \tag{1b}$$
$$V' = (T/m)\cos(\alpha + \delta) - D/m - g \sin\gamma, \tag{1c}$$
$$\gamma' = (T/mV)\sin(\alpha + \delta) + L/mV - (g/V) \cos\gamma. \tag{1d}$$

In Eqs. (1) appear the following quantities: longitudinal distance x, altitude h, velocity V, path inclination γ, mass m, acceleration of gravity g, thrust inclination angle δ, and angle of attack α. Also, the prime $'$ denotes derivative with respect to the actual time t. The thrust T, the drag D, the lift L, and the weight W are forces acting on the aircraft. These forces can be represented by the functional relations

$$T = T(h, V, \beta) \cong \beta\, T_{\max}(h, V), \tag{2a}$$
$$D = D(h, V, \alpha), \tag{2b}$$
$$L = L(h, V, \alpha), \tag{2c}$$
$$W = mg, \tag{2d}$$

where β is the thrust setting. Note that because of the small variations of the flight altitude, on this problem, the density ρ is imposed constant.

2.1. *Inequality Constraints.*

For several cases, initial numerical results have shown that the collision avoidance maneuver requires full thrust for the complete duration of the maneuver, which means we can enforce

$$\beta = 1\ . \tag{3}$$

To prevent the occurrence of the aerodynamic stall we want the angle of attack $\alpha(t)$ ranging between a minimum value and a maximum value

$$\alpha_{\min} \leq \alpha \leq \alpha_{\max}\,, \tag{4a}$$

moreover, to model the rotational inertia of the vehicle, we impose that the time rate $\alpha'(t)$, is subject to the similar constraint

$$-\,\alpha'_{\max} \leq \alpha' \leq \alpha'_{\max}\ , \tag{4b}$$

where $\alpha_{\max}$, $\alpha_{\min}$, $\alpha'_{\max}$ are prescribed positive constants. The above inequalities can be converted into equalities via the following nonsingular transformation:

$\alpha = (1/2)\,(\alpha_{max} + \alpha_{min}) + (1/2)\,(\alpha_{max} - \alpha_{min})\,\sin\eta$, (5a)

$\eta' = [2\alpha'_{max}/(\alpha_{max} - \alpha_{min})]\,\sin w$, (5b)

in which $\eta(t)$ denotes an auxiliary state variable and $w(t)$ denotes an auxiliary control variable. From Eqs. (5) we get

$\alpha' = \alpha'_{max}\cos\eta\,\sin w$, (6)

which implies that any pair of functions $\eta(t)$, $w(t)$ consistent with Eqs. (5) satisfies automatically the inequalities (4).

The transformation (5) has two advantages: (i) it is nonsingular; (ii) the angle of attack boundary is reached tangentially, regardless of the value of the auxiliary control. It is important to observe that these advantages are obtained at a price. In fact as the angle of attack moves toward its upper or lower boundary, the available α'-range shrinks proportionally to $\cos\eta$, vanishing at the upper boundary or lower boundary.

Eqs. (5), allow us to rewrite the system equation of the ekranoplan as

$x' = V\cos\gamma$, (7a)

$h' = V\sin\gamma$, (7b)

$V' = (\beta T_{max}/m)\cos(\alpha + \delta) - D/m - g\sin\gamma$, (7c)

$\gamma' = (\beta T_{max}/mV)\sin(\alpha + \delta) + L/mV - (g/V)\cos\gamma$, (7d)

$\eta' = [2\alpha'_{max}/(\alpha_{max} - \alpha_{min})]\,\sin w$, (7e)

where

$\alpha = (1/2)\,(\alpha_{max} + \alpha_{min}) + (1/2)\,(\alpha_{max} - \alpha_{min})\,\sin\eta$. (7f)

In the transformed system (7), the state variables are $x(t)$, $h(t)$, $V(t)$, $\gamma(t)$, $\eta(t)$; the new control variable is $w(t)$. Once $\eta(t)$ is determined, the original control $\alpha(t)$ can be recovered using Eq. (7f).

2.2. *Potential Collision.*

If the aircraft moves along a leveled trajectory with constant speed, its motion is described by the relation

$x(t) = x_0 + V_0\,t$, (8a)

$h(t) = h_0$, (8b)

where the subscript 0 denotes the quantities at the initial time $t = 0$. We assume that the obstacle is steady and located at the same initial altitude of the aircraft and at an initial distance d_0 from it

$x_*(t) = x_0 + d_0$, (9a)

$h_*(t) = h_0$. (9b)

Given the initial velocity of the aircraft V_0, and the initial distance of the obstacle d_0, the collision occurs if the following relations are satisfied at some forward time $\sigma > 0$:

$$x(\sigma) = x_*(\sigma) \, , \tag{10a}$$

$$h(\sigma) = h_*(\sigma) \, , \tag{10b}$$

with the implication that

$$V_0 \, \sigma = d_0 \, . \tag{11}$$

3. Optimization Problem.

For collision avoidance under emergency conditions, the best strategy is to maximize wrt the controls the timewise minimum distance between the aircraft and the obstacle. At the maximin point of the encounter, the distance between the ekranoplan and the obstacle has a minimum wrt the time, which occurs when the relative position vector is orthogonal to the relative velocity vector. In this way, we obtain an inner boundary condition to be satisfied at the maximin point separating the two main branches of the maneuver: the avoidance branch and the recovery branch. As consequence, a one-subarc Chebyshev problem can be transformed into a two-subarc Bolza problem solvable via the multiple-subarc sequential gradient-restoration algorithm (SGRA).

With the purpose to determine the inner boundary condition mentioned above, let the distance between the host aircraft and the obstacle be written as

$$d = \sqrt{\,[(x - x_*)^2 + (h - h_*)^2]} \tag{12a}$$

and its time derivative is

$$d' = (1/2) \, [(x - x_*) \, (x - x_*)' + (h - h_*) \, (h - h_*)']. \tag{12b}$$

Let also

$$D = (1/2) \, d^{\,2} = (1/2) \, [(x - x_*)^2 + (h - h_*)^2] \tag{13a}$$

denote the squared distance function whose time derivative is

$$D' = (x - x_*) \, (x - x_*)' + (h - h_*) \, (h - h_*)'. \tag{13b}$$

From (12b) and (13b) result that the conditions

$$d' = 0 \quad \text{and} \quad D' = 0 \tag{14a}$$

are reached simultaneously when

$$(x - x_*) \, (x - x_*)' + (h - h_*) \, (h - h_*)' = 0, \tag{14b}$$

that is, when (see Eqs. (8)-(9))

$$(x - x_*) \, V \cos \gamma + (h - h_*) \, V \sin \gamma = 0, \tag{14c}$$

namely, when the relative position vector is orthogonal to the relative velocity vector.

In the actual time domain, let $\theta_1 = \sigma$ denote the time length of the avoidance subarc; let $\theta_2 = \theta - \sigma$ denote the time length of the recovery subarc; let $\theta = \theta_1 + \theta_2$ denote the assigned time length of the entire maneuver. The two subarcs are connected to one another at the maximin point where the inner boundary condition (14c) is satisfied.

In order to bring the equations of the relative motion into the format required by the multiple-subar SGRA, it is necessary to normalize the interval of integration to unity. Having established that the time length of the first subarc is θ_1 and the time length of the second subarc is θ_2 , we introduce now a transformation from the actual time t to the virtual time τ rendering the virtual time length of each subarc equal to 1. More precisely, the time transformation is

$$\text{subarc } i = 1, \qquad \tau = t / \theta_1, \qquad 0 \le \tau \le 1, \qquad 0 \le t \le \theta_1, \tag{15a}$$

$$\text{subarc } i = 2, \qquad \tau = 1+(t - \theta_1)/ \theta_2, \qquad 1 \le \tau \le 2, \qquad \theta_1 \le t \le \theta. \tag{15b}$$

3.1. *Performance Index.*

For final time θ given, the optimization problem consists in maximizing wrt the new control $w(t)$ and parameter θ_1 , the distance function at the point where the inner boundary condition is satisfied.

In the actual time domain, the problem is

$$\max_{w(t),\, \theta_1} D(\theta_1) \quad \rightarrow \quad \min_{w(t),\, \theta_1} -D(\theta_1)\,. \tag{16a}$$

Let $F(t)$ be the one-index representation, in the actual time domain, of a generic function of the time F, and let $F(\tau, i)$ be its correspondent two-index representation in the virtual time domain, with i = 1 for the first subarc and i = 2 for the second subarc. In the virtual time domain, the problem becomes

$$\max_{w(\tau,i),\, \theta_1} D(1,1) \quad \rightarrow \quad \min_{w(\tau,i),\, \theta_1} -D(1,1)\,, \tag{16b}$$

with $D(1, 1)$ meaning $D(\tau, i)$ evaluated at the end (τ = 1) of the first subarc (i = 1). With the introduction of this new two-index representation the one-subarc Chebyshev problem has been replaced with a two-subarc Bolza problem.

3.2. *Penalized Performance Index.*

In order to prevent the undershooting of the initial altitude h_0 the performance index (16b) is replaced with the following penalized performance index:

$$\min\,[-D(1,1)+kP], \tag{17a}$$

where $k > 0$ is a suitable penalty constant and P is the penalty functional

$$P=\theta_1\int_0^1 E^2(\tau,\ 1)\mathrm{d}\tau+\theta_2\int_0^1 E^2(\tau,\ 2)\mathrm{d}\tau\,. \tag{17b}$$

In the above relation $E(\tau,\ i)$ measures the violation of the altitude threshold and is defined as follows:

$$E(\tau,\ i)=h_0-h(\tau,\ i)\,,\qquad \text{if}\quad h>h_0\,, \tag{17c}$$

$$E(\tau,\ i)=0\,,\qquad \text{if}\quad h\geq h_0\,. \tag{17d}$$

3.3. *Differential Constraints.*

Due to the normalization of the interval of integration, the equation of motion, can be rewritten as

$$\dot{x}=\theta_i\,[V\cos\gamma], \tag{18a}$$

$$\dot{h}=\theta_i\,[V\sin\gamma], \tag{18b}$$

$$\dot{V}=\theta_i\,[\,(\beta T_{\max}/m)\cos(\alpha+\delta)-D/m-g\sin\gamma], \tag{18c}$$

$$\dot{\gamma}=\theta_i\,[\,(\beta T_{\max}/mV)\sin(\alpha+\delta)+L/mV-(g/V)\cos\gamma], \tag{18d}$$

$$\dot{\eta}=\theta_i\,[2\alpha'_{\max}/(\alpha_{\max}-\alpha_{\min})\sin w], \tag{18e}$$

where the dot superscript denote a derivative wrt the virtual time τ, the index $i = 1,2$ denote the subarc, and

$$\alpha=(1/2)\,(\alpha_{\max}+\alpha_{\min})+(1/2)\,(\alpha_{\max}-\alpha_{\min})\sin\eta. \tag{18f}$$

3.4. *Boundary Conditions.*

The initial conditions are $x(0,1) = 0$, $h(0,1) = h_0$, $V(0,1) = V_0$, $\gamma(0,1) = 0$, $\eta(0,1) = \eta_0 = \sin^{-1}\{[2\alpha_0 - (\alpha_{\max} + \alpha_{\min})] / (\alpha_{\max} - \alpha_{\min})\}$, with h_0, V_0, γ_0, α_0 specified.

The continuity conditions at the interface, impose that the state variables at the end of the first subarc have the same values at the beginning of the second subarc. At the interface, the inner boundary condition is

$$\dot{D}\,(1,1)=0. \tag{19}$$

The final conditions are $h(1,2) = h_\theta$, $\gamma(1,2) = \gamma_\theta = 0$, with h_θ, γ_θ, specified.

3.5. *Bolza Problem.*

In conclusion, the Bolza problem of aircraft collision avoidance can be formulated as that of minimizing the performance index (17), subject to the differential constraints (18), the initial conditions, the continuity conditions, the inner boundary condition (22), and the final conditions.

With the actual final time θ given, the unknowns are the state and control variables of each subarc $x(t, i)$, $h(t, i)$, $V(t, i)$, $\gamma(t, i)$, $\eta(t, i)$, $w(t, i)$, plus the time parameter σ. With these functions known, the angle of attack $\alpha(t, i)$ of each subarc can be recovered via (18f).

4. Algorithm.

In this research, the multiple-subarc sequential gradient-restoration algorithm (SGRA) is applied to solve the collision avoidance problem involving an Ekranoplan and an obstacle located on the surface.

The multiple-subarc SGRA is an important extension of the single-subarc SGRA. The single-subarc SGRA was developed by Miele et al during the period 1968 to 1986. It has proven to be a powerful tool for solving optimal trajectory problems of atmospheric and space flight. Applications and extensions of this algorithm have been reported in the US, Japan, Germany, Spain, and other countries around the world; in particular, a version of this algorithm is used at NASA under the name SEGRAM, developed by McDonnell Douglas Technical Service (Ref. 14).

While the single-subarc SGRA deals with the optimization of a single system with initial and final boundary conditions, the multiple-subarc SGRA deals with the optimization of multiple systems with initial, final, and inner boundary conditions. In the multiple-subarc SGRA (see Refs. 2-3), a large and complicated overall system is decomposed into several subsystems along the time domain: each subsystem, having relatively simple properties, corresponds to a subarc; the connection between consecutive subsystems takes place via inner boundary conditions.

We note that, for stiff systems, it might be convenient to convert a single-subarc problem into a multiple-subarc problem with continuous inner boundary conditions. Indeed, proper increase of the number of subarcs can enhance considerably the robustness of the solution.

5. Data for Example.

The ekranoplan considered is an Alekseev A-90 Orlyonok powered, in cruise conditions, by a Kuznetsov NK-12MK turboprop engine with contra-rotating propeller.

The aerodynamic forces involved in the longitudinal dynamic are lift and drag, which are controlled via the angle of attack α. The angle of attack is bounded between the values α_{max} = 10 deg and α_{min} = -10 deg. To model the rotational inertia of the aircraft, the time rate of the angle attack is limited imposing α'_{max} = 3 deg/s. In ground effect flight, the aerodynamic coefficients are strongly dependent on the distance h between the ekranoplan and the water surface (Ref. 7).

The thrust expression considered for the turboprop engine is

$$T = \eta_P \Pi_{max} / V, \qquad 50 \text{ m/s} \le V \le 148 \text{ m/s}, \tag{20}$$

where V [m/s] is the velocity, η_P = 0.85 is the propeller efficiency, and Π_{max} = 15000 [hp] is the maximum engine power. Since the first runs, the results are such that the thrust value get its maximum value, for this reason we enforce that the avoidance maneuver and recovery maneuver are both operated at maximum thrust setting, $\beta = 1$.

At the beginning of the collision avoidance maneuver, the ekranoplan is in quasisteady leveled flight. Specifically, the initial conditions are $x_0 = 0$, h_0 = 2.5 m, V_0 = 105 m/s, $\gamma_0 = 0$, α_0 = 1.17 deg. After a transient period, at the end of the collision avoidance maneuver, the host aircraft recovers the leveled path. Specifically, the final conditions are h_θ = 2.5 m, γ_θ = 0 deg.

The obstacle initial distance d_0 is assumed ranging from 250 m to 1250 m. Namely we consider the following obstacle coordinates

$$250 \text{ m} \le x^* \le 1250 \text{ m}, \tag{21a}$$

$$h^* = 2.5 \text{ m}. \tag{21b}$$

6. Numerical Results.

The numerical computation of the optimal trajectories for collision avoidance was done using the multiple-subarc sequential gradient-restoration algorithm (SGRA). From initial results we found that, in any case, the avoidance maneuver tends to be performed with maximum thrust, this is why we impose the setting control at its maximum value $\beta = 1$. Under this assumption the only remaining control is the angle of attack $\alpha(t)$. The task of the multiple-subarc SGRA is to maximize, wrt the control $\alpha(t)$ and the time parameters σ, the timewise minimum distance between the ekranoplan and the obstacle: the vehicle is flying along a cruise leveled trajectory and the obstacle is steady on the sea surface. The numerical results are shown in Table 1 and Figures 1-2.

Table 1 show the effect of the initial distance d_0 on the maximin distance $d(\sigma)$. Also the correspondent maneuver time θ and the maximin time σ are reported. As the initial distance varies from 250 m to 1250 m, the maximin distance increases from 8.5 m to 370.7 m. In other words, an earlier detection of the obstacle will provide a larger safety margin for the ekranoplan.

Table 1. Effect of the initial distance on the maximin distance.

d_0 [m]	$d(\sigma)$ [m]	σ [s]	θ [s]
250	8.5	2.39	10
500	56.1	4.93	20
750	156.3	7.94	25
1000	297.0	11.91	30
1250	370.7	19.54	30

In Figs. 1-2 are presented the main results of the collision avoidance maneuver for an initial distance d_0 = 1250 m. Fig. 1 shows the collision avoidance trajectory. Fig. 2 reports the time history of the control $\alpha(t)$.

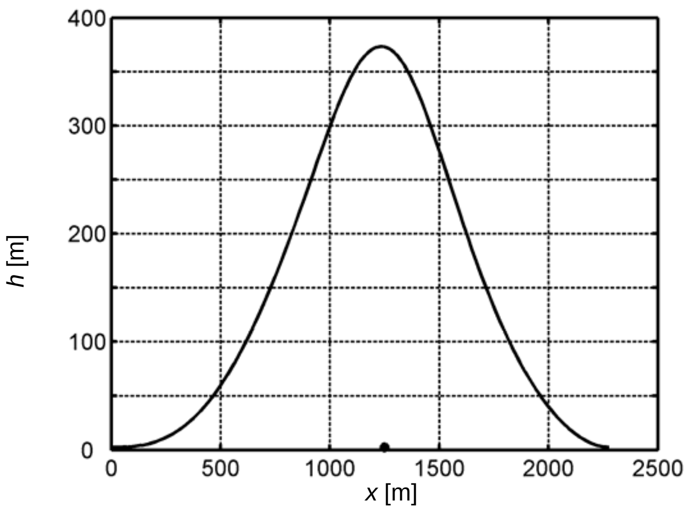

Figure 1. Optimal trajectory, d_0 = 1250 m.

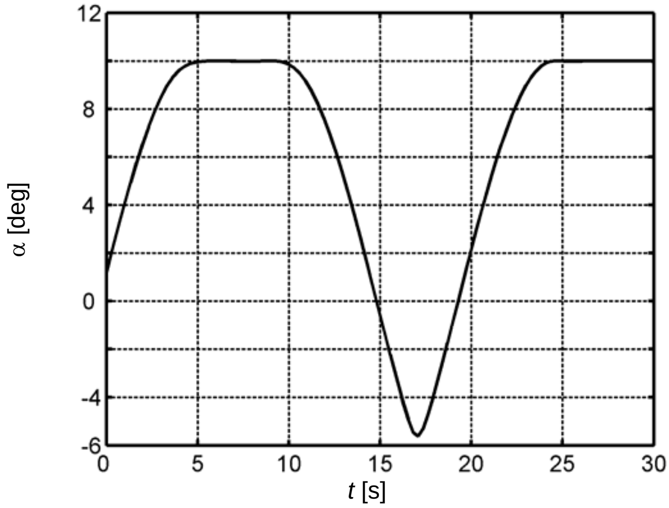

Figure 2. Time history of the angle of attack, d_0 = 1250 m.

6. Conclusions.

In this work we investigate the optimal collision avoidance problem between a cruising Ekranoplan and a steady obstacle located on the ground. The following assumptions are employed: (i) initially the Ekranoplan is moving in a quasisteady levelled cruise trajectory; (ii) after the avoidance manoeuvre a recovery manoeuvre is executed by the aircraft to return to the initial cruise path; (iii) both the avoidance manoeuvre and the recovery manoeuvre lie on the same vertical plane identified by the initial cruise trajectory. The investigation of the collision avoidance performances on the longitudinal flight is encouraged by the relatively large quasi-flat turn radius of the Ekranoplans.

The results show that the angle of attack changes with the maximum speed. Eventually, for the cases with longer initial distances from the obstacle, the positive angle of attack bound is reached, meaning the ekranoplan tends to reach the stall conditions. The characteristics of the control that we have found, make it particularly suitable for guidance purposes.

In this initial phase of our research we have tested the goodness of the approach employed. A three-dimensional avoidance trajectory should be investigated to compare jumping performances vs. quasi-flat turning performances.

REFERENCES

1. Miele, A., and Wang, T., *Optimal Collision Avoidance in Aerospace under Emergency Conditions,* Paper IAC-05-C1.5.01, 55th International Astronautical Congress, Fukuoka, Japan, 2005.
2. Miele, A., and Wang, T., *Multiple-Subarc Sequential Gradient-Restoration Algorithm, Part 1: Algorithm Structure,* Journal of Optimization Theory and Applications, Vol. 116, No. 1, pp. 1-17, 2003.
3. Miele, A., and Wang, T., *Multiple-Subarc Sequential Gradient-Restoration Algorithm, Part 2: Application to a Multistage Launch Vehicle Design,* Journal of Optimization Theory and Applications, Vol. 116, No. 1, pp. 19-39, 2003.
4. Rishikof, B. H., McCormick, B. R., Pritchard, R. E., and Sponaugle, S. J., *SEGRAM: A Practical and Versatile Tool for Spacecraft Trajectory Optimization*, Acta Astronautica, Vol. 26, Nos. 8-10, pp. 599-609, 1992.
5. Miele, A., and Wang, T., *Maximin Approach to the Ship Collision Avoidance Problem via Multiple-Subarc Sequential Gradient-Restoration Algorithm*, Journal of Optimization Theory and Applications, Vol. 124, No. 1, pp. 29-53, 2005.
6. Miele, A., and Wang, T., *Optimal Trajectories and Guidance Schemes for Ship Collision Avoidance,* Journal of Optimization Theory and Applications, Vol. 129, No. 1, 2006.
7. Gatto, C., *Study on the Dynamic Behavior of an Ekranoplan,* Master Thesis, University of Palermo, 2004.
8. Frazzoli, E., Mao, Z. H., Oh, J. H., and Feron, E., *Resolution of Conflicts Involving Many Aircrafts via Semidefinite Programming,* Journal of Guidance, Control, and Dynamics, Vol. 24. No. 1, pp. 79-86, 2001.
9. Menon, P. K., Sweriduk, G. D., and Sridhar, B., *Optimal Strategies for Free-Flight Air Traffic Confliction Resolution,* Journal of Guidance, Control, and Dynamics, Vol. 22. No. 2, pp. 202-211, 1999.
10. Clements, J. C., *The Optimal Control of Collision-Avoidance Trajectories in Air-Traffic Management*, Transportation Research, Vol. 33B, No. 4, pp. 265-280, 1999.
11. Nocedal, J., and Wright, S.J, *Numerical Optimization*, Springer Series in Operations Research, Springer Verlag, New York, NY, 1999.