

Blind Source Separation with Sparsity Constraints for Magnetoencephalography

Massimo Fornasier

Dip. di Metodi e Modelli Matematici per le Scienze Applicate
Università di Roma "La Sapienza"
Via A. Scarpa 16, 00161 Roma, Italy
fornasier@dmmm.uniroma1.it

Francesca Pitolli

Dip. di Metodi e Modelli Matematici per le Scienze Applicate
Università di Roma "La Sapienza"
Via A. Scarpa 16, 00161 Roma, Italy
pitolli@dmmm.uniroma1.it

Vittorio Pizzella

Dip. di Scienze Cliniche e Bioimmagini
Università di Chieti-Pescara
Via dei Vestini 33, 66013 Chieti Scalo, Italy
pizzella@unich.it

1 Introduction

Several observed phenomena are a suitable superposition of basic events. To understand the core of such phenomena it is fundamental to identify the sources of these basic events and their contribution to the overall phenomena.

A classical example is the cocktail party problem, that is the problem to separate the voices of several people speaking simultaneously in the same room, using recordings of few microphones. Similar events arise in many fields, such as telecommunications, image analysis, biological signal processing. In particular, we are interested in biomedical signal processing where the data, originating from electrical or magnetic field measured by means of an array of suitable sensors, as in Electroencephalography (EEG) or Magnetoencephalography (MEG), is used to assess the underlying electric currents.

Blind Source Separation (BSS) techniques are successfully used in this context. Under the assumptions that there exists a *linear dependence* between the observed signals \mathbf{f} and the source activity signals \mathbf{u} , and the sources are stochastic processes with prescribed statistical properties, BSS aims at identifying the unique linear transformation K such that $K\mathbf{f} = \mathbf{u}$, where \mathbf{u} are the unique sources with the expected statistical properties. "*Blind*" refers to the lack of any assumptions on the geometrical distribution of the sensors or the underlying sources.

BSS has been successfully associated with Independent Component Analysis (ICA) where one assumes that the sources are statistically independent. Although this technique has shown to be extremely efficient for a huge number of new applications in BSS, in several concrete applications the independence assumption is unrealistic. An example of this is functional brain mapping, that is the problem of identifying the different regions of the brain where spontaneous or stimulated activity takes place.

Here, we want to investigate a new generation of methods for BSS. These methods, recently introduced in [5], are based on the so-called *sparsity assumption*: the source signals are assumed to be represented as a sum of weighted basic signals belonging to a prescribed *dictionary* \mathcal{D} , where only few of them are relevant. Since the sparsity assumption depends on the dictionary, one can also design the dictionary in order to obtain the most sparse representation. Also for this reason, the sparsity assumption is often more realistic than the sole statistical independence. Examples of dictionaries that arise sparse representations of several natural signals (e.g., natural images, audio signals, biosignals) are wavelets [4] and curvelets [2].

2 Blind Source Separation with Sparsity Constraints

Let $\mathbf{f} = L\mathbf{u} + \mathbf{n}$ be the observed signal; L is a known linear operator from the source signal space X into the observed signal space Y , and \mathbf{n} is a statistical noise. The source separation problem under the sparsity constraints consists in minimizing the functional

$$(2.1) \quad J_{\Psi}(\mathbf{u}) := \|L\mathbf{u} - \mathbf{f}\|_Y + \Psi[(\mathbf{u}_{d^i})_{i=1, \dots, M, d^i \in \mathcal{D}}],$$

where $\mathbf{u} := \sum_{i=1}^M \sum_{d^i \in \mathcal{D}} \mathbf{u}_{d^i} d^i$ and Ψ is a *suitable sparsity measure*. The dictionary $\mathcal{D} = \{d^i\}$ is designed to let the solution $\mathbf{u}^{(\infty)}$ of the minimum problem be sparsely represented. The minimization algorithm consists in constructing a sequence of minimizers of certain approximating functionals for which the individual minimization is easier. To prove convergence of the minimizers, a rather general theory of variational convergence is required.

3 Magnetoencephalography

Aim of MEG is the analysis of brain functionality through the measurements of the tiny magnetic fields generated by neuronal currents [6].

Compared to other techniques, such as fMRI (functional Magnetic Resonance Imaging) or PET (Positron Emission Tomography), MEG directly senses neuronal functions with a millisecond time resolution. Moreover, MEG is a noninvasive technique. For this reason MEG technology appears particularly attractive. The neuromagnetic field is very weak, i.e. only in the order of 10^{-13} Tesla in magnitude, that is several orders smaller than the earth's steady magnetic field or other magnetic noise sources.

For this reason, to successfully resolve the current density flowing within the brain, it is mandatory to use low noise superconducting magnetometers as well as sophisticated signal processing. Presently available devices are able to sample the magnetic field in about 150-300 points all over the head, with a sampling space of few centimeters. Current density is estimated from magnetic field using a suitable model.

A simple but effective model for the basic neuromagnetic source is the current dipole inside a spherical homogeneously conducting medium [6]. Under the hypothesis that the returning Ohmic currents are negligible, the magnetic field generated outside a homogeneously conducting sphere Ω , can be written following the Biot-Savart law as

$$(3.1) \quad \vec{B}_{\vec{q}}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\vec{q}(\vec{x}) \times (\vec{r} - \vec{x})}{|\vec{r} - \vec{x}|^3} d\vec{x}.$$

The function \vec{q} represents the spatial dipole momentum density at the position r and μ_0 is magnetic permeability of the vacuum space.

4 A model of BSS for MEG

In this section, we focus on the design of optimized algorithms based on the sparsity assumption for the estimation and removal of the noise and on the development of new BSS methods for MEG data. As shown in [1], the sparsity constraints for modeling brain activity seems to be realistic. We formulate the MEG problem as the minimizer of the functional J_Ψ , when L is the Biot-Savart operator. In particular, for a given set of magnetic field measurements $\{m_1^*, \dots, m_M^*\}$, we want to determine a configuration of the spatial dipole momentum density \vec{q} that minimizes the discrepancy $|m_j(\vec{q}) - m_j^*|, j = 1, \dots, M$, where $m_j(\vec{q})$ is the normal component of the magnetic field $\vec{B}_{\vec{q}}(\vec{q})$ measured at r_j .

It is clear that this is a strongly ill-posed problem, especially if the measurements are considered affected by (high) noise. Therefore, regularization is required and performed by introducing into the minimization further constraints on the solution. By the nature of the problem, we can expect that the dipole momentum density \vec{q} is only locally distributed in the sphere Ω , so that it can be certainly assumed to be piecewise smooth, where the support is relatively localized and models certain active areas of the brain. This allows us to incorporate a sparsity constraints into the model. Since the Biot-Savart operator (3.1) is a well defined operator on $C^\infty(\Omega; \mathbb{R}^3)$, it is bounded in the $L^2(\Omega; \mathbb{R}^3)$ -norm [3, Th. D]. Therefore, the minimization fits with the methods described in §2 and, in particular, with that in [5] where the minimizing algorithm is proved to be convergent.

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