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Mimetic finite difference methods for convection-dominated problems

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The Mimetic Finite Difference Method (MFDM) is a a relatively new technique [4] that has already been applied to the solution of problems of continuum mechanics, electromagnetics, gas dynamics and linear diffusion.

The idea behind MFDM is to define the discrete operators by imposing that the essential properties of the underlying differential operators are preserved. As a numerical technique, it may be classified as standing in between Mixed Finite Element Methods and Finite Volumes.

Consider, for example, the linear diffusion problem written as a system of two equations:

$$\operatorname{div} \boldsymbol{F} = b, \quad \boldsymbol{F} = -K \operatorname{grad} p$$

where K is generally a full symmetric tensor. The first equation represents mass conservation while the second one is the constitutive equation relating the pressure p to the velocity field $\boldsymbol{F}$. The discrete (mimetic) differential operators div and grad are defined so that the Gauss divergence theorem holds on the discrete pressure and velocity spaces. In this way, local mass conservation is embedded in the method.

Another crucial property of MFDM is that very general polyhedral mesh elements can be handled [5],[1],[2]. The convergence analysis presented in [1] imposes a very mild restriction on the geometry of the mesh elements, allowing for non-convex and degenerate polyhedrons. Such analysis has been recently extended to include even polyhedrons with curved faces [3].The flexibility in the mesh design gives an obvious advantage in the treatment of complex solution domains and heterogeneous materials. Moreover, allowing non-matching, non-convex mixed types of elements facilitate adaptive mesh refinement, particularly in the coarsening phase, making it a completely local process.

The talk will overview the features of MFDM. We shall demonstrate through extensive numerical examples the flexibility of the method and discuss its superconvergence properties. The main topic of the talk is a new extension of the method to the stable solution of convection-dominated diffusion problems, exemplified by the model equation

$$\nabla \cdot (-K \nabla p + \beta p) = b,$$

where β is a given velocity field. The problem is again written as a first-order system by defining a velocity variable. This can be either the full flux $-K\nabla p + \beta p$ or simply the diffusion part of the flux $-K\nabla p$.In this second case, we show that a conservative full flux can be reconstructed. The convergence properties of various stabilization techniques will be discussed and compared through numerical examples.

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