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On Multilane Traffic Flow

Rinaldo M. Colombo
Dipartimento di Matematica - Università di Brescia
Via Branze 38 I-25123 Brescia
`rinaldo.colombo@ing.unibs.it`

Andrea Corli
Dipartimento di Matematica - Università di Ferrara
Via Machiavelli 35 I-44100 Ferrara
`andrea.corli@unife.it`

We consider continuum models for multilane traffic flow consisting essentially on a system of balance laws coupled only through suitable source terms. In other words, the convective part of each equation forming the system describes the *intra–lane* dynamics; on the contrary, each right hand side models the interplay between adjacent lanes.

As a first example consider the following model for n lanes proposed in [9]:

$$\left\{\begin{array}{rcl}\partial_t\rho_1 + \partial_x(\rho_1 v_1) & = & \dfrac{\rho_2}{T_2^1} - \dfrac{\rho_1}{T_1^2} \\ \partial_t\rho_j + \partial_x(\rho_j v_j) & = & \dfrac{\rho_{j-1}}{T_{j-1}^j} - \dfrac{\rho_j}{T_j^{j-1}} + \dfrac{\rho_{j+1}}{T_{j+1}^j} - \dfrac{\rho_j}{T_j^{j+1}} \\ \partial_t\rho_n + \partial_x(\rho_n v_n) & = & \dfrac{\rho_{n-1}}{T_{n-1}^n} - \dfrac{\rho_n}{T_n^{n-1}}\,. \end{array}\right. \tag{0.1}$$

Here, the subscripts 1, $j = 2,\ldots,n-1$ and n refer to the lane numbers. The quantities ρ_ℓ, respectively $v_\ell = v_\ell(\rho_\ell)$, are the vehicle density, respectively the average traffic speed, in the ℓ-th lane, for $\ell = 1,\ldots,n$; at last $T_j^k = T_j^k(\rho_j,\rho_k)$ is the transition rate from lane j to lane k, with $j-k=\pm1$.

In this example the intra-lane dynamics is described by a LWR *scalar* equation. More general n-lanes models can be obtained by describing the dynamics in each lane with more general single-lane models, for instance using a 2×2 system of conservation laws, also called a *"higher order"* model. Typical examples are the models proposed by Aw and Rascle [1] or Colombo [4]. It may be also reasonable to use different analytical descriptions for the dynamics of the different lanes, thus mixing scalar equations and systems.

All the situations described above fit within a unique analytical framework, provided by the class of systems

$$\partial_t u_\ell + \partial_x\left[f_\ell(u_\ell)\right] = g_\ell(t,x,u) \qquad \ell = 1,\ldots n \tag{0.2}$$

where each state variable u_ℓ varies in $\mathbb{R}^{m_\ell}$. The above system is studied in [7].

However, traffic flow models usually display further structure. First, the interaction among lanes reduces to that relating each lane to its neighbor(s). This amounts to require, for instance, that the coupling source terms satisfy

$$g_\ell(t,x,u) = g_{\ell-1}^{\ell}(u_{\ell-1},u_\ell,u_{\ell+1}) - g_\ell^{\ell+1}(u_\ell,u_{\ell+1},u_{\ell+2})\,,\quad g_{-1}^0 = g_n^{n+1} = 0\,, \tag{0.3}$$

where $g^{\ell}_{\ell-1}$ describes the amount of vehicles that pass from lane ℓ to lane $\ell-1$ per unit time. Second, the conservation of $\sum_\ell \int u_\ell \, dx$ follows from

$$\sum_{\ell=1}^{n} g_\ell = 0\,. \tag{0.4}$$

We stress that the conservation of the "momentum variable" in higher order models is often not required. Hence, in these models, the above sum is restricted to the source terms in the density equations. We recall here that the introduction of source terms in the momentum equations can used to simulate various realistic situations, see [3, 2]. In particular, in the example (0.1) we have $u_\ell = \rho_\ell$ and both conditions (0.3) and (0.4) are required.

Below, we extend the results in [5, 7] to prove the well posedness of the initial value problem for (0.2) under the key assumption that the homogeneous part of the system is of Temple type. We point out that this condition is satisfied in all the cited models.

The present analytical framework applies also to the case of various additional source terms, for example those describing entries, exits or inhomogeneities (ascents, descents) in the road, see [3].

It is reasonable to postulate that the lane changing rate at a point $\bar{x}$ depends on a space average of the traffic density before $\bar{x}$. This leads to consider source terms satisfying (0.3)-(0.4) but involving a causal convolution kernel. We will show therefore an extension of the recent result [8] that ensures the well posedness of these nonlocal multilane traffic models. This result allows, for instance, to use the techniques in [6] to estimate *a posteriori* the various characteristic parameters.

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