# Performace prediction of olympic rowing boats accounting for full dynamics

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# 1 Overview

The periodic forces imposed at the oars and due to the movement of the rowers induce a secondary motion on the scull, causing an additional drag which may represent a significant part of total dissipated energy. We have taken the approach of computing the complete scull motion including pitching, vertical and horizontal movement. A full dynamic model requires simulating rowers inertial forces, thrust forces at the oarlocks and fluid-dynamic forces. It is a complex fluid-structure interaction problem that we analyzed by coupling different fluid dynamic models with a dynamic model of the boat.

# 2 Reference frames

We denote with  $(\mathbf{O}; X, Y, Z)$  the global (inertial) reference frame, and with  $\mathbf{e}_X$ ,  $\mathbf{e}_Y$  and  $\mathbf{e}_X$  the corresponding versors. The X axis is directed horizontally and points towards the bow, being aligned with the mean velocity of the boat. The Z axis is directed vertically pointing upwards, while the water free surface is located at  $Z = h^0$ , where  $h^0$  is a constant value representing the undisturbed water level. Since only the motion in the (X, Z) plane is studied, all the considered forces lie in this plane.

We also introduce a relative reference frame  $(\mathbf{G}_c; x, y, z)$ , attached to the boat hull (supposed to be a rigid body) and centered in its baricenter  $\mathbf{G}_c$ . The x, y, z axes versors in this frame of reference will be  $\mathbf{e}_x$ ,  $\mathbf{e}_y$  and  $\mathbf{e}_z$ .

With these assumptions, the pitch angle  $\phi$  is the angle between  $\mathbf{e}_X$  and  $\mathbf{e}_x$ , and is positive when the bow lowers. Once we have introduced the rotation matrix

(2.1) 
$$\mathcal{R}(\phi) = \begin{bmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{bmatrix},$$

we can write the coordinate transformation law for a generic point P

(2.2) 
$$\begin{bmatrix} R_X^P \\ R_Y^P \\ R_Z^P \end{bmatrix} = \mathcal{R}^T(\phi) \begin{bmatrix} r_x^P \\ r_y^P \\ r_z^P \end{bmatrix} + \begin{bmatrix} G_X^c \\ G_Y^c \\ G_Z^c \end{bmatrix}$$

where positions in the global system are denoted by capital letters.

Transformations between velocity and acceleration vectors in the two reference frames assume the form

(2.3) 
$$\mathbf{V}^P = \dot{\mathbf{P}} = \mathbf{v}^P + \dot{\mathbf{G}}^C + \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{G}^C),$$

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#### A. Mola et al.

(2.4) 
$$\mathbf{A}^{P} = \ddot{\mathbf{P}} = \mathbf{a}^{P} + \ddot{\mathbf{G}}^{C} + \dot{\boldsymbol{\omega}} \times (\mathbf{P} - \mathbf{G}^{C}) + \boldsymbol{\omega} \times \boldsymbol{\omega} \times (\mathbf{P} - \mathbf{G}^{C}) + 2\boldsymbol{\omega} \times \mathbf{v}^{P},$$

being  $\boldsymbol{\omega} = \dot{\phi} \, \mathbf{e}_Y$  the angular velocity vector. Here, the dot symbol denotes time derivatives.

#### 3 Dinamic system and governing equations

We assume now that the motion of the rowers in the relative reference frame is assigned, being  $\mathbf{g}^{v^i} = \mathbf{g}^{v^i}(t) = (g_x^{v^i}(t), g_z^{v^i}(t))$  the motion law for the baricenter of the *i*-th rower, that can be recast in the absolute reference frame by means of transformation (2.2). We can finally write the motion equations for a system composed by scull, oar, and oarsmen, namely

(3.1a) 
$$M\ddot{\mathbf{G}}^{c} + \left(\mathcal{O}(\phi)\sum_{i=1}^{n}M^{v^{i}}\mathbf{g}^{v^{i}}\right)\ddot{\phi} = -2\dot{\phi}\mathcal{O}(\phi)\sum_{i=1}^{n}M^{v^{i}}\dot{\mathbf{g}}^{v^{i}} - \dot{\phi}^{2}\mathcal{R}^{T}(\phi)\sum_{i=1}^{n}M^{v^{i}}\mathbf{g}^{v^{i}} - \mathcal{R}^{T}(\phi)\sum_{i=1}^{n}M^{v^{i}}\ddot{\mathbf{g}}^{v^{i}} + \mathcal{R}^{T}(\phi)\sum_{i=1}^{n}\mathbf{f}^{r^{i}}(t) + M\mathbf{g} + \mathbf{F}^{a},$$

(3.1b) 
$$\left(\mathcal{R}^{T}(\phi)\sum_{i=1}^{n}M^{v^{i}}\mathbf{g}^{v^{i}}\right) \times \ddot{\mathbf{G}}^{c} + \left(I_{y}^{c} + \sum_{i=1}^{n}M^{v^{i}}||\dot{\mathbf{g}}^{v^{i}}||^{2}\right)\ddot{\phi} = -2\dot{\phi}\sum_{i=1}^{n}M^{v^{i}}\mathbf{g}^{v^{i}}\cdot\dot{\mathbf{g}} - \sum_{i=1}^{n}M^{v^{i}}\mathbf{g}^{v^{i}}\times\ddot{\mathbf{g}} + \sum_{i=1}^{n}M^{v^{i}}(G_{X}^{v^{i}} - G_{X}^{c})g + M^{a}.$$

Here g is the module of gravity acceleration (9.81  $m/s^2$ ),  $M^{v^i}$  is the mass of the *i*-th rower,  $I_y^c$  is the moment of inertia of the boat around the y axis, while the matrix  $\mathcal{O}(\phi)$  is defined as

(3.2) 
$$\mathcal{O}(\phi) = \frac{d}{d\phi} \mathcal{R}^T(\phi) = \begin{bmatrix} -\sin\phi & \cos\phi \\ -\cos\phi & -\sin\phi \end{bmatrix}$$

We now have a system of three second order ordinary differential equations in the time variable, in which  $\mathbf{u} = [\mathbf{G}_X^c(t), \mathbf{G}_Z^c(t), \phi(t)]$  is the unknown vector, its components being the position of the scull center of gravity  $\mathbf{G}^c(t)$  and the pitch angle.

To close the problem, however, we must determine the values of the traction forces applied on each of the oars —namely  $\mathbf{f}^{r^i}(t)$ — and the forces and moments acting on the hull, due to its interaction with the surrounding water.

## 4 Oars traction forces and forces due to the hull interaction with water

The former kind of forces can be computed by analyzing the dynamics of the oar itself. Assuming a rigid oar having negligeble mass, we can write, composing the linear and angular (around the oarlock) momentum conservation law

(4.1) 
$$\mathbf{f}^{r^{i}}(t) = \frac{r_{b}}{L_{r}}\mathbf{f}^{s^{i}}(t)$$

being  $L_r$  the total lenght of the oar, and  $r_b$  the distance bethween the rower's hands and the oarlock. The oarlock forces  $\mathbf{f}^{s^i}(t)$  can be measured by means of suitable sensors placed in the oarlock, and are therefore assigned.

The hydrostatic and hydrodynamic forces and moments are decomposed in the following way

(4.2) 
$$\mathbf{F}^{a} = S^{a}\mathbf{e}_{Z} - R^{a}\mathbf{e}_{X} + \mathbf{D}^{a},$$
$$M^{a} = M^{a}_{S} + M^{a}_{D}.$$

Here  $S^a$  and  $M_S^a$  are the hydrostatic lift and moment respectively, and depend on the instantaneous position of the hull. The drag due to the primary motion  $R^a$  is computed by means of the formula

$$R^a = \frac{1}{2}\rho S_{ref} C_{dX} (\dot{G}_X^c)^2,$$

being  $S_{ref}$  a reference surface and  $C_{dX}$  a drag coefficient, computed for each boat, performing a Navier–Stokes simulation of the stationary motion.

Finally, the forces and moments due to the secondary motions of the boat, namely  $\mathbf{D}^a$ and  $M_D^a$ , are computed by solving a suitable elliptic partial differential problem (see [2]) for the complex velocity potential  $\Psi_s = \alpha_s + i\beta_s$  where  $\alpha_s$  and  $\beta_s$  are two scalar functions representing the velocity potential and the stream function.

It turns out that these forces present a component proportional to the acceleration vector  $\ddot{\mathbf{u}}$  — the mass matrix  $\mathcal{M}$ — and a component proportional to the velocity vector  $\dot{\mathbf{u}}$  —the damping matrix  $\mathcal{S}$ .

Introducing these quantities in equations (3.1) we get a system of the form

(4.3) 
$$A(t, \mathbf{y}(t))\frac{d\mathbf{y}}{dt}(t) = \mathbf{B}(t, \mathbf{y}(t)), \quad t > 0$$

where  $\mathbf{y} = [\mathbf{G}_X^c(t), \mathbf{G}_Z^c(t), \phi(t), \dot{\mathbf{G}}_X^c(t), \dot{\mathbf{G}}_Z^c(t), \dot{\phi}(t)]$ . Employing  $\mathbf{y}$  instead of  $\mathbf{u}$  leads to a first order ODE system, instead of a second order one. This allows the use of several numerical schemes developed for this kind of problems. In particular, we employed schemes included in GSL libraries (see [1]).

# 5 Results

The algorithm here illustrated has been implemented in a C++ language code. A typical solution for problem (4.3) is depicted in Fig. 5, representing a time hystory plot for each component of **y** vector. The algorithm proves to be robust, returning physically

#### A. Mola et al.

correct results for any crew configuration tested. Still, improvements have to be made in several areas.

# REFERENCES

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