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One-dimensional Global Optimization Problems with Multiextremal Constraints[1]

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In this talk, the following one-dimensional global optimization problem is considered: to find the value f^* and a point x^* such that

$$f^* = f(x^*) = \min\{f(x) : x \in [a, b],\ g_j(x) \le 0,\ 1 \le j \le m\} \tag{0.1}$$

where $f(x)$ and $g_j(x)$, $1 \le j \le m$, are multiextremal Lipschtiz functions (particularly, this means that they can be non-differentiable), i.e.,

$$|\, g_j(x') - g_j(x'')\,| \le L_j\ |\, x' - x''\,|,\ \ x', x'' \in Q_j,\ \ 1 \le j \le m+1, \tag{0.2}$$

$$0 < L_j < \infty, \quad 1 \le j \le m+1. \tag{0.3}$$

(in order to unify the description, the designation $g_{m+1}(x) \triangleq f(x)$ is used hereafter).

It is also supposed that the objective function $f(x)$ and the constraints can be partially defined. This means that a constraint $g_{j+1}(x)$ is defined only at subregions where $g_j(x) \le 0$ and $f(x)$ is defined only over subregions of $[a, b]$ where all the constraints are satisfied.

It should be noted that not only problem (0.1) but univariate global optimization problems in general (in contrast to one-dimensional local optimization problems that have been very well studied in the past) continue to attract attention of many researchers

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(see, e.g., [7, 11, 18] and the references given therein). These happens at least for two reasons. First, there exists a large number of applications where it is necessary to solve such problems: electrical engineering and electronics are among the fields where one-dimensional global optimization methods can be used successfully (see, e.g., [3, 12, 18]). Second, there exist numerous approaches (see, e.g., [5, 7, 8, 10, 11, 15, 18, 19, 20]) enabling to generalize to the multidimensional case the methods proposed for solving univariate problems.

It is not easy to find an efficient algorithm for solving problem (0.1). For example, the penalty approach requires that $f(x)$ and $g_i(x)$, $1 \le i \le m$, are defined over the whole search interval $[a, b]$. At first glance it seems that at the regions where a function is not defined it can be simply filled in with either a big number or the function value at the nearest feasible point. Unfortunately, in the context of Lipschitz algorithms, incorporating such ideas can lead to infinitely high Lipschitz constants, causing degeneration of the methods and non-applicability of the penalty approach.

A promising approach called the *index scheme* has been proposed in [16] (see also [1, 14, 17, 18]) in combination with information stochastic Bayesian algorithms for solving problem (0.1)–(0.3). An important advantage of the index scheme is that it does not introduce additional variables and/or parameters by opposition to classical approaches in [2, 7, 8, 9]. It has been recently shown in [13] that the index scheme can be also successfully used in combination with the Branch-and-Bound approach if the Lipschitz constants $L_j, 1 \le j \le m+1$, from (0.2), (0.3) are known a priori.

If there exists an additional information allowing us to obtain a priori fixed constants K_j, $1 \le j \le m+1$, such that

$$L_j < K_j < \infty, \quad 1 \le j \le m+1,$$

then the algorithm IBBA from [13] can be used. However, in practical applications the Lipschitz constants $L_j, 1 \le j \le m+1$, are very often unknown. Thus, the problem of their estimating arises inevitably.

In this talk, new geometric methods using adaptive estimates of Lipschitz constants are described and its convergence conditions are established. Particularly, algorithms with the local tuning technique on behaviour of both the objective function and constraints are considered. Results of numerical experiments on a set of non-differentiable test functions from [4] (available also at http://wwwinfo.deis.unical.it/∼yaro/constraints.html) including comparison of the proposed algorithms with both methods using penalty approach and the IBBA are presented. They demonstrate the advantage of the new methods with respect to the traditional ones in terms of the performed function/constraints evaluations.

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