

## Variational Models for Traffic Equilibrium Problems

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Equilibrium is a central concept. Variational inequalities (VI) and generalized variational inequalities (GVI) provide a powerful framework for the study of equilibrium problems, both from a theoretical and from a computational point of view. We focus on finite-dimensional VI and its applications to equilibrium problems and we show the relationships between VI and optimization problems. Emphasis is then given on algorithms for the computation of equilibria, along with theoretical convergence analysis. In particular, we draw our attention to equilibrium problems in transportation networks. The traffic network equilibrium problem forms integrated part of transportation planning process: this problem provides estimated future traffic distribution and allows to evaluate what effects changes in the traffic network infrastructure will have on the network's performance. The variational approach allows us to model traffic equilibrium problems successfully, in both the static and the dynamic case and in both the fixed and the variable travel demand case. We concentrate on the static traffic equilibrium model with fixed demand. The equilibrium conditions can be given in terms of the flows on paths in the network (path formulation) as well as in terms of multicommodity flows on arcs, where each commodity represents one O/D pair (arc formulation). We believe the formulation with path-flow variables to be more appealing for several reasons. A major one is that solving the traffic assignment in the path flow space provides automatically, not only the equilibrium flow, but also all the routed paths. Another important reason for using path flow variables, is the possibility to determine, with a required high accuracy, the equilibrium solution. We present a new efficient procedure for solving the traffic equilibrium problem. Since we operate in the space of path flows, the specific features of the traffic equilibrium problem leads to use projection-type methods. Moreover, to overcome the difficulty of enumerating all the path connecting an O/D pair, we algorithmically generate, for each O/D pair, only those paths which potentially will carry a positive flow in an equilibrium solution (column generation approach). To show the efficiency of the proposed algorithm we present the results obtained performing numerical experiments on several traffic networks. Computational results suggest that the proposed path-based algorithm is a valuable alternative to arc-based algorithms in order to obtain an accurate solution of the traffic assignment problem for large scale networks.

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