G-Index Numbers of the Production of Pollutions, the Linked Social Impact and Non Measurable Variations

Numeri Indice generalizzati (g-IN) della produzione di sostanze inquinanti, del relativo impatto sociale e variazioni non misurabili

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Riassunto: L'obiettivo del lavoro è la semplificazione della comunicazione dei dati ambientali attraverso l'uso di indicatori. Gli indicatori ambientali si classificano in base allo schema Pressione Stato Risposta (DPSIR). Nel lavoro ci si occupa di *indicatori di impatto* legati ad inquinanti il atmosferici (PM_{10}). La strategia di ricerca seguita è basata sulla Teoria dei Numeri Indice generalizzati (g-IN) introdotta da Verrecchia (2003, 2005). L'utilizzo dei g-IN in ambito ambientale ha il vantaggio di trattare i *"missing values"* e le variazioni non misurabili, oltre che considerare gli effetti sociali dell'inquinamento. Per le applicazioni è stata utilizzata una serie storica giornaliera di rilevazioni degli inquinanti atmosferici in provincia di Milano messa a disposizione dall'Agenzia Regionale per la Protezione dell'Ambiente (ARPA) della Lombardia.

Keywords: Numeri Indice generalizzati (g-IN), indicatori di impatto ambientale.

1. Introduction and Aims

The aim of this work is to introduce the generalized Index Number (g-IN), and its application in the field of atmosphere pollution. The g-IN application presented is built on the available data i.e. the daily data of PM_{10}, provide from ARPA, and the monthly data of population, provide from ISTAT (from 1-11-03 to 1-07-2004).

2. Methods

In order to account for the pollution data structure (missing values), we need to generalize the usual number index formulas.

- **Definition 1:** *The indicator function common to both situations*

Let $I_{[0],z}$ (with z = 1,2,…,Z) be the Z indicator functions in 0 of the z^{-th} value. Let $I_{[t],z}$ be the Z indicator functions in t of the z^{-th} value. A non-negative application

$$ {}^{c}I_{[0\cap t],z} = I_{[0],z} \cdot I_{[t],z} \qquad (z = 1,2,\ldots,Z) \qquad (1) $$

is called indicator function common to both situations[1] (in 0 and t). •

([1]) Where $I_{[0],z} = \begin{cases} 1 \text{ for } v_{00,z} > 0 \\ 0 \text{ for } v_{00,z} = 0 \end{cases}$ and $I_{[t],z} = \begin{cases} 1 \text{ for } v_{tt,z} > 0 \\ 0 \text{ for } v_{tt,z} = 0 \end{cases}$. And where $v_{00,z} = p_{0,z}\, q_{0,z}$ and $v_{tt,z} = p_{t,z}\, q_{t,z}$. And where $p_{.,z}$ is the pollution (PM_{10}) and where $q_{.,z}$ is, for example, the population weight.

• **Definition 2:** *The g-Laspeyres aggregative index number formulae*

Let ${}_0P_t^{g\text{-}L}{}_{RS}{}_{[l]}$ (with l = 1,2,…,L) be the g-Laspeyres index number[2] of the l^{-th} land. Let ${}^cV_{00,l}$ be the sum by z of the r<Z values in 0 (where z = 1 , 2, … , r, … , Z) common to both situations (0 and t) of the l^{-th} land. A non-negative application

$$ {}_0P_t^{g\text{-}L}{}_{RS}{}_{[\cup_L l]} = \sum_{l=1}^{L} {}_0P_t^{g\text{-}L}{}_{RS}{}_{[l]} \frac{{}^cV_{00,l}}{\sum_{l=1}^{L} {}^cV_{00,l}} \qquad (2) $$

is called aggregative formulae of g-Laspeyres index number. •

• **Definition 3:** *The g-Paasche aggregative index number formulae (*g-Laspeyres cofactor*)*

Let ${}_0Q_t^{g\text{-}P}{}_{RS}{}_{[l]}$ (with l = 1,2,…,L) be the g-Paasche index number[3] of the l^{-th} land. Let ${}^cV_{t0,l}$ be the sum by z of the r<Z product of pollution in t and population in 0 (where z = 1 , 2, … , r, … , Z) common to both situations (0 and t) of the l^{-th} land. A non-negative application

$$ {}_0Q_t^{g\text{-}P}{}_{RS}{}_{[\cup_L l]} = \sum_{l=1}^{L} {}_0Q_t^{g\text{-}P}{}_{RS}{}_{[l]} \frac{{}^cV_{t0,l}}{\sum_{l=1}^{L} {}^cV_{t0,l}} \qquad (3) $$

is called aggregative formulae of g-Paasche index number. •

• **Definition 4:** *The impact index common to both situation* A non-negative application

$$ {}^c{}_0V_t = {}_0P_t^{g\text{-}L}{}_{RS}{}_{[\cup_L l]} \; {}_0Q_t^{g\text{-}P}{}_{RS}{}_{[\cup_L l]} \qquad (4) $$

is called impact index common[4] to both situations. •

([2]) Where ${}_0P_t^{g\text{-}L}{}_{RS}{}_{[l]} = \dfrac{\sum_{z=1}^{Z} p_{t,z}\, q_{0,z}\, {}^cI_{[0\cap t],z}\, I_{[l],z}}{\sum_{z=1}^{Z} p_{0,z}\, q_{0,z}\, {}^cI_{[0\cap t],z}\, I_{[l],z}}$ and $I_{[l],z}$ is the l^{-th} indicator function of the land.

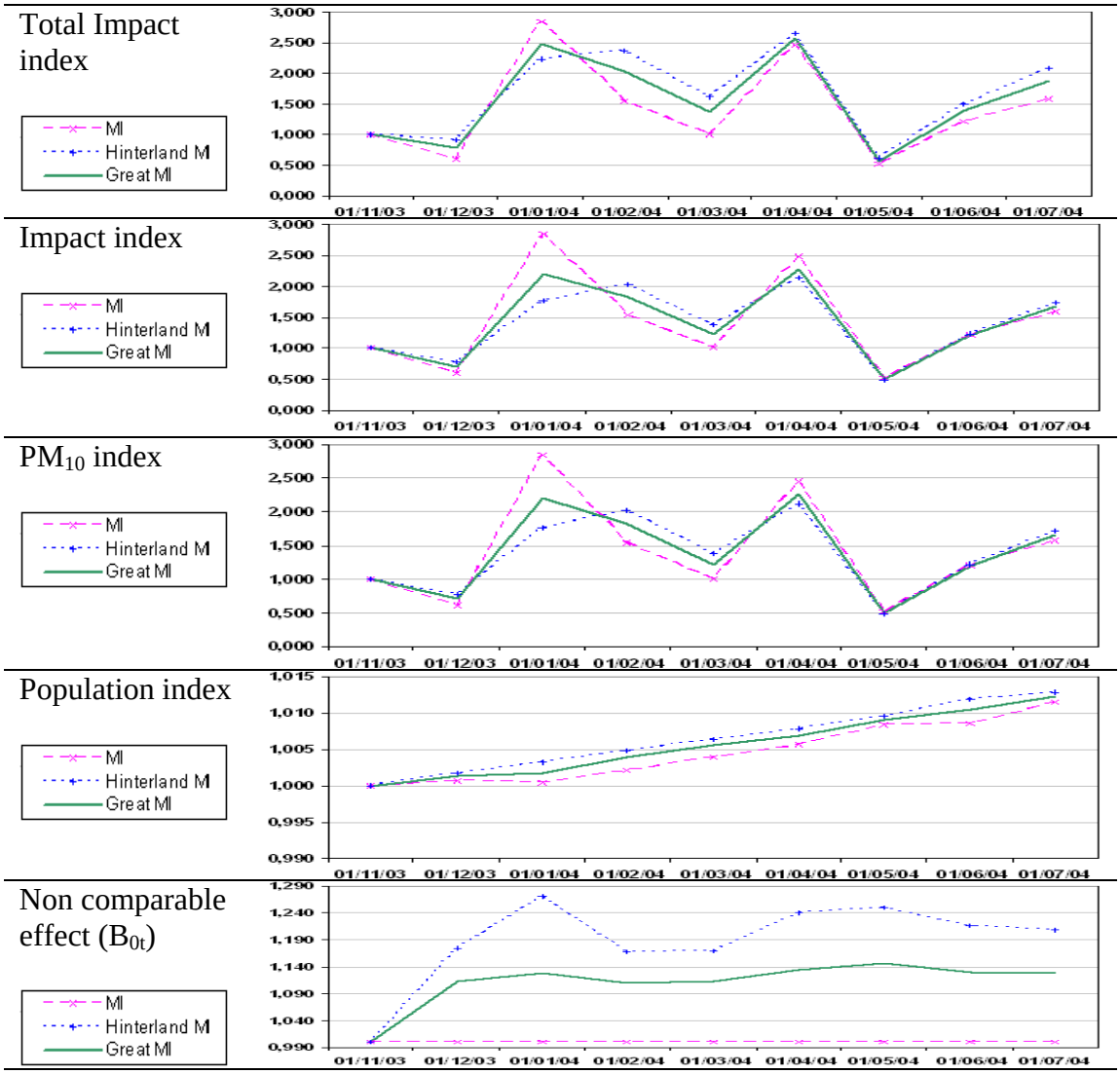
([3]) Where ${}_0Q_t^{g\text{-}P}{}_{RS}{}_{[l]} = \dfrac{\sum_{z=1}^{Z} p_{t,z}\, q_{t,z}\, {}^cI_{[0\cap t],z}\, I_{[l],z}}{\sum_{z=1}^{Z} p_{t,z}\, q_{0,z}\, {}^cI_{[0\cap t],z}\, I_{[l],z}}$ and $I_{[l],z}$ is the l^{-th} indicator function of the land.

([4]) The total impact index (${}_0V_t$) can be factorized on contribution due to the impact index common to both

3. Application

The application consist in a Aggregative g-index number system. The impact index common to both situations factorizations (g-indices) are able to explain variations like contribution due to pollutions and to population of the impact index (Figure 1.). The data used, in this first stage, are daily for the PM_{10} (provide from ARPA), and monthly from the population (provide from ISTAT). Particularly data from 1-11-03 to 1-07-2004 have been employed. The great Milan impact index have been built from the PM_{10} and Population g-index. And both from the sub-g-indices of Milan and Hinterland. By this application is easy to extend the same methodology to the European area and sub area without to restart from the data.

Figure 1: *Aggregative g-IN system: Milan, Hinterland of Milan, Great Milan*



Source: ARPA Lombardia; ISTAT [On line], available: http://demo.istat.it.
Note 1. Population sub-area is a crude estimation.

situations and to Basket factor (Verrecchia 2003, 2004b, 2005): ${}_0\mathbf{V}_t = {}^c{}_0\mathbf{V}_t \cdot \mathbf{B}_{0t}$.

4. Result and Conclusion

The generalized index number (g-IN) theory provide the impact index common to both situations and, as its factorizations, g-indices able to explain variations like contribution due to pollutions and to population. Table 1. shows the index of great Milan that has been built from Milan g-IN and Hinterland of Milan g-IN. So, from set of sub-area, it is possible to build g-indices of sub-g-indices, if it is provided the correct weight and, obviously, the sub-g-indices.

Table 1: *Great Milan aggregative g-IN: from Milan g-IN & Hinterland of Milan g-IN*

1-7-04/1-11-03	MI	$Weight_{MI}$	Hinterland MI	$Weight_{HintMI}$	Great MI
Total Impact index (note 4)	1,5895	-	2,0904	-	1,8872
Impact index (expr. 4)	1,5895	-	1,7313	-	1,6737
PM_{10} index (expr. 2) ($w={}^{c}V_{00}/\sum {}^{c}V_{00}$)	1,5714	0,4058	1,7092	0,5942	1,6533
Population index (expr. 3) ($w={}^{c}V_{t0}/\sum {}^{c}V_{t0}$)	1,0115	0,3857	1,0129	0,6143	1,0124
B_{0t} - Non comparable effect (note 4)	1,0000	-	1,2075	-	1,1275

Source: ARPA Lombardia; ISTAT [On line], available: http://demo.istat.it.

The methodology is available. There are problems about data quality and data production to build impact index: the population data are not provided daily, like pollution one, so daily impact index are not generally calculable without the use of estimation techniques.

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