DOI: 10.1478/AAPP.99S1A23

AAPP | Atti della Accademia Peloritana dei Pericolanti
Classe di Scienze Fisiche, Matematiche e Naturali
ISSN 1825-1242

Vol. 99, No. S1, A23 (2021)

STUDY OF THE OSCILLATIONS OF DROPS OF NEWTONIAN LIQUIDS INDUCED BY ACOUSTIC VIBRATIONS

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ABSTRACT. The aim of this paper is to verify the applicability of the Rayleigh-Lamb equation to drops having different diameters and, specifically, with diameters of the order of magnitude of centimeters (macro drops), millimeters or micrometers (micro drops). On this purpose three experiments have been taken into account. A completely new fact is constituted by the following question: it is possible to apply the Rayleigh-Lamb equation to a heterogeneous drop like that represented by a soap bubble, characterized by a surface tension different from the substance constituting the drop's mass (air)? The results reported in this paper seem to confirm that this is possible. The order of magnitude of the calculated autofrequencies is comparable to that observed experimentally. The limitations of the experiments are the geometry of the system. The Rayleigh-Lamb equation applies, strictly speaking, to a free drop not subjected to the action of external forces. This would be possible through the use of special devices, i.e the Acoustical Levitated. Therefore, it was decided to carry out the experiments with drops bound by viscous adhesion to the respective supports. It is evident that the geometry of the drop is no longer perfectly spherical, however the results obtained do not seem to have suffered greatly from this limitation.

1. Introduction

This research was born in the 2018-2019 school year in the Physics (Magazù *et al.* 2012; Castorina *et al.* 2018; Caccamo *et al.* 2019) course during the study of forced oscillations and mechanical resonance. As is known, each oscillating system has its own frequency f_0 which depends on the geometric and physical characteristics of the system itself (Keiji Kanazawa and Gordon 1985; Cannon 2003). If the system is subjected to periodic stresses produced by an external force (henceforth forcing) of frequency f, then it can be demonstrated, by solving the relative differential equations, that the amplitude of the forced oscillations is given by:

$$x = \frac{x_0 \cos \omega t}{1 - r^2} \tag{1}$$

Having indicated with r the ratio f/f_0 , with $\omega = 2\pi f$ the pulsation of the forcing and with x_0 the elongation of the system in static regime (system subjected to a force F of constant intensity with $f = 0$).

A detailed analysis under the mathematical profile of the relationship (1) is beyond the scope of this research. What interests us here is that for values of r very close to 1 (*i.e.*, when the forcing frequency is very close to the system's own frequency), the amplitude of the oscillation is enhanced (at the limit for $r = 1$, theoretically $x \to \infty$).
That said, it should be noted that this paper is inspired by previous experimental work carried out at the Clamson University of South Caroline and which is briefly described here:

- A droplet of water with a diameter of about 0.5 mm (ejected from the needle of a syringe) is levitated and oscillated in an acoustic field produced by an ultrasound source (Caccamo *et al.* 2017; Cannuli *et al.* 2018b,c);
- The geometry of the droplet takes on a flat shape, which is closely related to the harmonics of the acoustic vibration (Caccamo and Magazù 2016; Cannuli *et al.* 2018a; Cannuli and Caccamo 2019);
- At a given harmonic (8^{th}) the elastic response of the droplet (ie the surface tension value) is no longer able to follow the external oscillations of the acoustic field and therefore the droplet "shatters" into smaller fractions, remaining however in a liquid state.

Hence our investigation:

a) Is a drop of liquid similar to an oscillating system?
b) If it is, what are the limits of applicability of the results deduced from the oscillation mechanics?
c) What possible practical applications?

2. Project description - research areas

As far as question a) is concerned, we went in search of the literature on what is already available on the subject. Our attention has focused on the Rayleigh-Lamb solution (Rogers 1995; Banerjee and Kundu 2006), of which we give a summary:
The incompressibility of the volume of a liquid and the elasticity of its surface allow a wide range of harmonic behaviors. For example, a drop of water imagined for spherical simplicity and on which external forces do not act, will present, for what has been said, resonant frequencies, corresponding to the possible stationary waves that can be generated on its surface, supported by the recall action elastic surface tension. This set of normal ways was first studied by Lord Kelvin in the late 1800s and later by other illustrious physicists, such as Rayleigh, Lamb and Chandrasekhar.
Rayleigh-Lamb's solution to the problem of the autofrequencies of a free drop, which allows to determine both the fundamental frequency (or mode) and the subsequent harmonics is as follows

$$f_n = \frac{1}{2\pi}\sqrt{\frac{n(n+2)(n-1)\gamma}{3\pi\rho V}} \tag{2}$$

where: n is the natural integer > 1, ρ is the density of the liquid, γ is the surface tension and V is the drop volume. Equation 2 takes a simpler form if we assimilate the geometry of the drop to that of a sphere of radius R and therefore of volume $(4\pi R^3)/3$, substituting one has:

$$f_n = \frac{1}{4\pi^2}\sqrt{\frac{n(n+2)(n-1)\gamma}{\rho R^3}} \tag{3}$$

Is to be noted the perfect formal and dimensional analogy of (2) with the best known formula which gives the proper frequency of a mass-spring oscillator $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where k is the elastic constant of the system and m the mass of the system.
In equation 2, the elastic constant is represented by the surface tension γ which provides elastic response of the droplet to the periodic stresses of the forcing, while the product ρV is precisely the mass of the drop. Of course, a linear mass-spring system has only one degree of freedom and therefore only one way of oscillating, that is, only one frequency of its own. In (2) the factor $n(n+2)(n-1)$ justifies the fact that a free droplet in three-dimensional space has more degrees of freedom and therefore more ways of oscillating when n changes (Reid 1960; Alleyne 1991; Temperton 2013; Cogo 2015). We now analyze (b) the limits of applicability of equation (3). When you think of a droplet, you usually imagine it with a diameter limited to a few millimeters. But, theoretically, nothing prevents us from speculating on a droplet of much larger diameter (of the order of centimeters) or much smaller (of the order of microns).
So the experimental challenge is: how can we verify the applicability of this relationship to drops of centimetre diameter (macro drops), millimetre or micrometric diameter (micro drops)?
The first challenge required very special experiments and, if it is also allowed, completely original.

3. Description of the experiments

3.1. Harmonic oscillations of macro drops (soap bubbles). As we have clarified previously, (3) applies to a spherical droplet of radius R on which no external forces act. In the case of a droplet that must have a diameter of the order of centimeters, it is certainly a big problem to achieve the required boundary conditions. Can you imagine a drop of water with a diameter of 5 cm (for example) suspended in the air? Not only that, but there is also the problem of finding a source that can act as a forcing.
Then it was considered appropriate to make the following hypothesis: what if you tried it with soap bubbles? We agree that a soap bubble is not a homogeneous drop, but why not try to verify if (3) can also be applied to a macroscopic drop of density equal to that of air and with a surface tension equal to that of water soapy? We sincerely have not encountered conceptual difficulties in a hypothesis of applicability of the above formula. Of course it is a bit simpler now but some technical problems still need to be resolved.

3.2. Production of soap bubbles. A common kit for the production of bubbles can be good for obtaining soap bubbles of various diameters (it depends, as many know, on how much air you blow inside). To extend the average life of a soap bubble and make it more resistant to oscillations, we have added glycerin and sugar (Caccamo and Magazù 2017; Magazù *et al.* 2018).

3.3. Measurement of the diameter of a soap bubble. In (3) the radius of the drop is shown. In our case the radius of the soap bubble. How to measure it without breaking it? Obviously you cannot use the centesimal caliper, or even think of other methods that require contact with the bubble. So we thought of an optical method. Once a bubble has been obtained, it is picked up on the support (see Fig.1) and put in position. A linear spot laser beam (obtained with a glass cylinder placed at the exit of the beam) can slide on a metal bench.

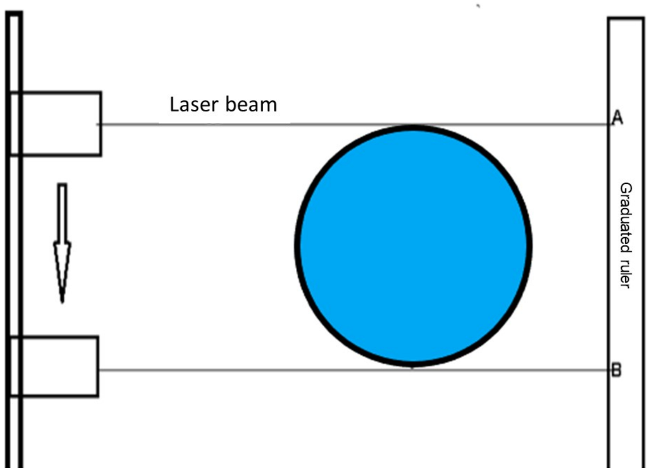

FIGURE 1. Device for measuring the diameter of a soap bubble.

When the beam touches the bubble, you are sure that it is the maximum diameter (point A) precisely for the geometry of the beam. An optical refraction phenomenon warns us that the beam has touched the end of the bubble (A) and so also occurs in point B. By looking at A and B on a ruler it is possible to measure by difference the value of the diameter and therefore the radius of the bubble. Obviously everything must be done very quickly, taking into account the relatively short life of the soap bubble. We need a lot of caution and above all a lot of patience.

3.4. Resonance oscillations. Once the value of R has been obtained, from a sheet of excel prepared for the calculation, the value of the fundamental frequency and then of the higher harmonics is immediately obtained $(n = 2, 3, 4, \dots)$ (Caccamo and S. 2016; Caccamo *et al.* 2018; Caccamo and Magazù 2018; Magazù and Caccamo M. 2018). A default value is set in the generator by default at f_0 and the frequency is gradually increased until the bubble oscillations (Plesset and Prosperetti 1977; Benjamin 1989; Dawson 2002; Kracht and Finch. 2009) are obtained in the resonance regime and subsequently the resonance for the upper harmonics (Fig. 2).

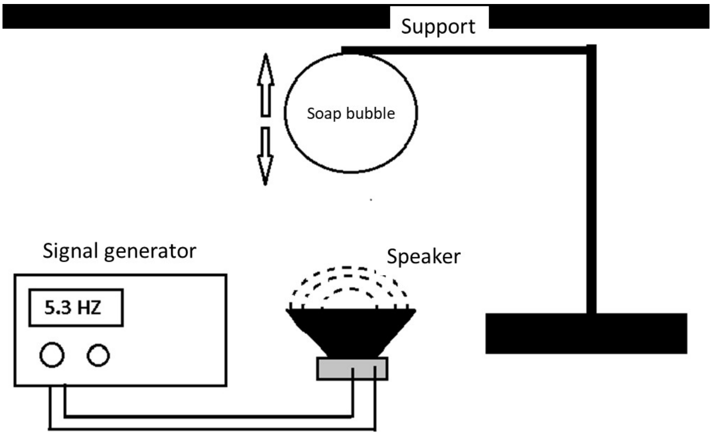

FIGURE 2. Method used for the measurement of resonance oscillations.

It is observed that:

- the soap bubble does not oscillate for all the forcing frequencies. When approaching the fundamental frequency, the amplitude begins to increase and for the critical value, in proximity of f_0, the amplitude increases clearly: there is mechanical resonance;
- Once the fundamental frequency value has been exceeded, the oscillations decrease significantly until they resume at the second harmonic $(n = 3)$ and so on;
- In correspondence with the subsequent harmonics, the amplitude of the oscillations is reduced compared to that relating to the fundamental frequency and this for the obvious reason that a greater number of elastic waves must be confined in the same spherical surface. That is, the same phenomenon occurs in an elastic cord fixed at its ends when stationary waves are generated inside it.

The graph of the function $f_0 = f_0(D)$ is shown in the figure 3, with a diameter having the range $3 - 10cm$.

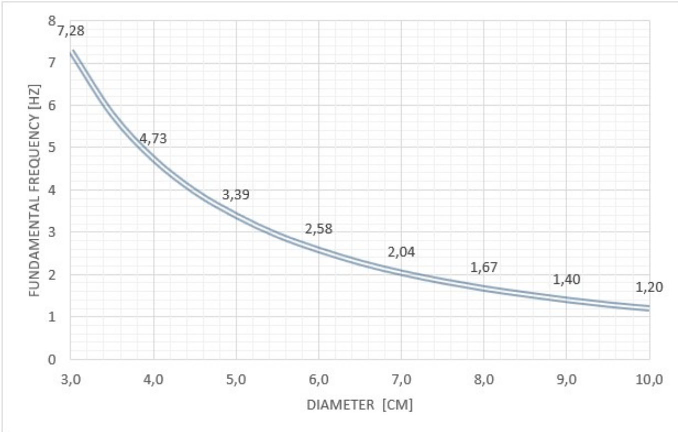

FIGURE 3. Graph of the function $f_0 = f_0(D)$ with a diameter having the range $3 - 10cm$.

All tests carried out with bubbles of various centimetric diameter give values that agree with the expected values.

4. Analysis of the experiments

4.1. Resonance oscillation of a soap bubble with a diameter of about 6 cm. To obtain a soap bubble resistant to the pressures induced by acoustic vibrations, capable of guaranteeing a video recording, we had to create a mixture of soapy water, sugar and glycerin. For a value of the surface tension calculated with the Du Nouy ring, of $\gamma \simeq 0,021N/m$ and for a radius of $3cm$, a fundamental frequency of $1.82Hz$ is obtained from (3). For $n = 5$ the resonance frequency is $7.62Hz$. In our experiment, the frequency at which the resonance oscillation is observed is $.60Hz$, which we verified to be the fifth harmonic, in good agreement with the (3).

4.2. Oscillations of millimetre drops. In this section we want to show if equation 2 is valid for millimeter droplets. But first we have to solve some technical problems.
Find a system to swing a drop of a few millimeters in diameter using the same sound source as a force. It was tried with a drop of water suspended from an orifice but it was too heavy and not affected by the forcing. It has been tried with a drop of ethyl ether which is lighter, with a density of $700K/mc$. In this case the droplet fluctuates.
Inserting in equation 3 the values relating to a drop of ethyl ether with a diameter of $2mm$ we have for the fundamental frequency:

$$f_0 = \frac{1}{4\pi^2}\sqrt{\frac{8 \cdot 0.016N/m}{700Kg/m^3 \cdot (1 \cdot 10^{-3})^3 m^3}} \cong 10.7Hz \tag{4}$$

In the experiment it is observed that a droplet of ethyl ether hanging from the orifice of a syringe, subjected to harmonic oscillations, begins to oscillate in a resonance regime precisely for frequencies very close to the calculated one. The figure 4 shows the graph of the resonance frequency for a drop of ethyl ether as the diameter changes.

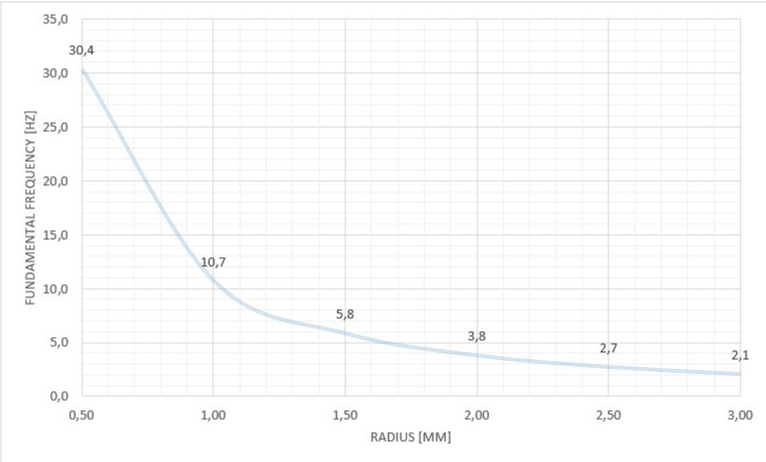

FIGURE 4. Graph of the resonance frequency for a drop of ethyl ether as the diameter changes.

4.3. Harmonic oscillations of micrometric droplets. In this case, we want to extend, theoretically, the field of application of (3) drops of water with a micrometric diameter $(1\mu = 10^{-6}m)$. If in 3 we try to give the parameters the values relating to a drop of water with a diameter of 1μ, we obtain, for the fundamental frequency $(n = 2)$:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{n(n+2)(n-1)\gamma}{3\pi\rho V}} = 0.612MHz \qquad (5)$$

where: $\gamma = 73x10^{-3}N/m$ surface tension of distilled water; $\rho = 1000Kg/m^3$ density of distilled water.

For subsequent values of n (see Table 1) the self-frequencies are obtained:

TABLE 1.

n	f
3	**1,18 Mhz**
4	1,83 Mhz
5	2,55 Mhz
6	**3,35 Mhz**
7	4,54 Mhz
8	5,11 Mhz

We are in the ultrasound field. How to test the validity of (2) in this case? There would be one way: to use electromedical ultrasonic sources, those used by physiotherapists for

the treatment of muscle trauma. In particular, two sources were used, one at $1Mhz$ and the other at $3Mhz$.
Using water as a liquid, it is observed that:

- At $1Mhz$ the ultrasonic waves generate a "cavitation" effect on the air-water interface. Having exceeded a critical distance between the ultrasonic probe and the free surface of the water, cavitation generates oscillations which produce micro-droplets of fog;
- At a frequency of about $3Mhz$ the micro droplets of fog oscillate in resonance and implode. The forcing, at that frequency, manages to break the surface tension and the imploding drops pass directly from the liquid phase to the vapor phase. What is observed in the experiment is that the fog is swept away by the ultrasonic wave;
- The phenomenon is observable for small depths of the thickness of fog.

The $1Mhz$ source initially produces a curious effect probably due to cavitation. As the ultrasonic source moves away from the water-air interface, a "suction effect" is created which cannot depend on the viscous adhesion of the water to the metal plate of the ultrasonic source. Increasing the distance further generates the mist of water droplets. If at this point an ultrasonic wave at $3Mhz$ is launched into the fog layer, a resonance is obtained for which the vibrations break the surface tension of the droplet of fog which, by implosion, passes directly from the liquid to the vapor phase , having the effect of dispersing the fog.
This type of system, having sources with higher frequencies available, could be a valuable aid in cases where weather conditions increase the genesis of fog, a highly risky condition especially for means of transport (eigenvector, trains, aircraft, etc. . .).

5. Conclusions

The experiments performed, even within the limits of the experimental errors due to:

- Imprecision of the measures
- System geometry not perfectly spherical
- Lack of time to carry out a wider sampling

they allow us to have reasonable confidence in the fact that the ways of oscillating a drop of liquid are linked to the same fundamental relationship.
In particular, it would be extremely interesting to investigate whether there is a critical value of the resonance frequency at which the implosion of the drop occurs and to deepen this analysis from the point of view of the energies involved.

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Paper contributed to the international workshop entitled “New Horizons in Teaching Science”, which was held in Messina, Italy (18–19 november 2018), under the patronage of the *Accademia Peloritana dei Pericolanti*

Manuscript received 15 September 2020; published online 30 September 2021