

Refining Characterization of Powder for Rotomolding Industries: A Study Case

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Introduction

Powder, in general, is characterized by a number of *macroscopic* parameters. From among them one is *granulometry*, which represents the mean of the random variable: *particle weight of size x* ; ($0 < x < \infty$). In this study it is shown how determining the mathematical form of the probability density, from which this numerical value comes. This function allows, in turn, defining the probability density of particle number, included in a prefixed weight of material. This *microscopic* know how can help predicting more accurately the actual powder quality. The experimental data, used in this work, come from a leading industry, in the field of plastic rotational moulding. The firm buys this raw material from an external supplier. Even though the methodology, used to obtain the results shown below, is valid in general terms, and applicable to different contexts, the numerical outcomes, obviously, fully apply only to those industries, which use plastic powders, similar to that considered in this work.

1. Particle weight probability density

Figure 1 shows schematically some particles, with a graph, provided by supplier, which indicates some weight fraction versus their size.

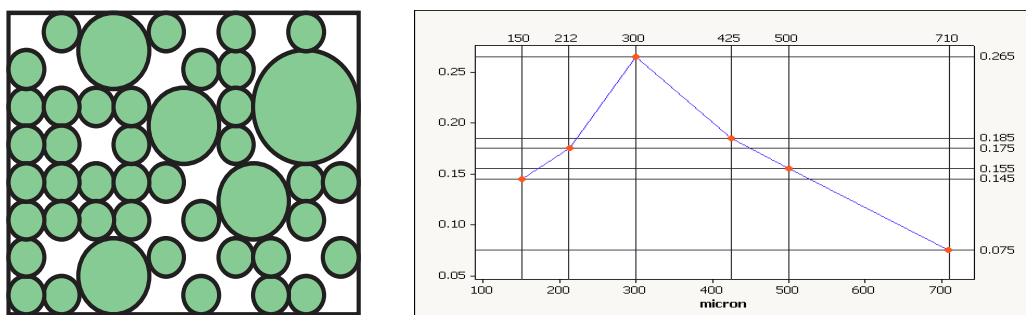


Fig.1 - A box of particles with a graph describing weights versus size

In order to find the particle density function, the following steps have been performed: 1) *Interpreting the exact meaning of the graph of figure 1*; it has been understood that the ordinate of 300μ , for example, is the weight fraction of particles with size included in the range $212 \mu - 300 \mu$; with this in mind, it has been possible to: 2) *Calculating some ordinates of density function*; this graph is reported in figure 2, below; for example, the 181μ density ordinate comes from figure 1, in this way: $0.002822 = 0.175 / (212 - 150)$ 3) *Formulating a conjecture on the theoretical parent density*; this has been the most delicate task; 4) *Verifying the conjecture, using the data points of figure 1*; (102 graphs of this kind were available; the conjecture has been tested on 6 of those,

spread in the course of 3 years); 5) *Defining the mathematical form of density, based on the conjecture.* From a careful visual inspection of density shape belonging to the six lots, it was conjectured that *the weight density was that belonging to a mixture of two normal random variables*; the path, to reach this conclusion, is illustrated below, using the numerical values of the graph of figure 2. The conjecture, in this case, was that *two normal random variables, $N(\mu_1 = 220; \sigma_1 = 85)$, of density $f_{1,x}$, and $N(\mu_2 = 450; \sigma_2 = 65)$, of density $f_{2,x}$ were mixed.* Stated that, it was calculated the percent, say $P_1\%$, of the first one. The obtained value was 66.8%, coming from equation: $P_1 = h_1 / f_{1,181} = 0.002822 / 0.004224$, where $h_1 = 0.002822$, is the ordinate, read in figure 2, of $x = 181\mu$ and $f_{1,181} = 0.004224$ is the density value of the random variable, $N(\mu_1 = 220; \sigma_1 = 85)$, corresponding to $x = 181\mu$. The density value of $x = 181\mu$, of the second random variable, was disregarded, because numerically negligible (0.0000012). Said that, the conjecture was experimentally tested, using the data of figure 1.

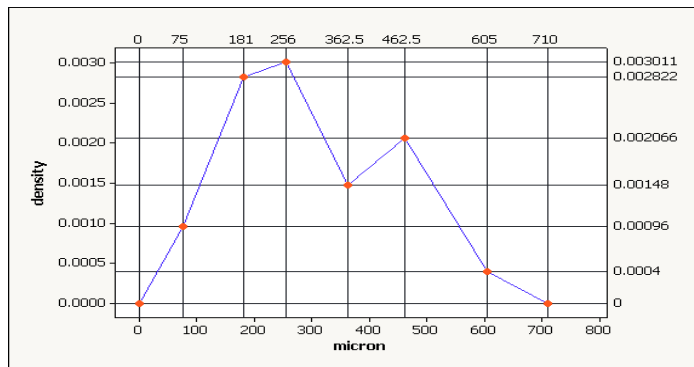


Fig. 2 – Experimental particle weight probability densities

Being, in general terms:

$$f_{1,x} = \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} \quad ; \quad f_{2,x} = \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} \quad (1)$$

Putting:

$$[f_1 + f_2]_x = P_1 f_{1x} + (1 - P_1) f_{2x} ; [N_1 + N_2]_a^b = P_1 \int_a^b f_{1x} dx + (1 - P_1) \int_a^b f_{2x} dx \quad (2)$$

12 different independent tests were possible. The obtained results are shown below. Numerical values, calculated, through the equations of the two random variables are put within square brackets, those available experimentally, and visible on the graphs of figures 1 and 2, are put within round brackets.

$$\begin{aligned} [f_1 + f_2]_{75,0} &= [0.001090] \leftrightarrow (0.000960) ; [f_1 + f_2]_{181,0} = [0.002822] \leftrightarrow (0.002822) \\ [f_1 + f_2]_{256,0} &= [0.002890] \leftrightarrow (0.003011) ; [f_1 + f_2]_{362,5} = [0.001592] \leftrightarrow (0.001480) \\ [f_1 + f_2]_{462,5} &= [0.002054] \leftrightarrow (0.002066) ; [f_1 + f_2]_{605,0} = [0.000357] \leftrightarrow (0.000400) \end{aligned}$$

$$\begin{aligned}
[N_1 + N_2]_{10}^{150} &= [0.1370] \leftrightarrow (0.1450) ; [N_1 + N_2]_{150}^{212} = [0.1720] \leftrightarrow (0.1750) \\
[N_1 + N_2]_{212}^{300} &= [0.2467] \leftrightarrow (0.2650) ; [N_1 + N_2]_{300}^{425} = [0.2019] \leftrightarrow (0.1850) \\
[N_1 + N_2]_{425}^{500} &= [0.1687] \leftrightarrow (0.1550) ; [N_1 + N_2]_{500}^{710} = [0.0727] \leftrightarrow (0.0750)
\end{aligned}$$

Furthermore, two independent numerical values of powder *granulometry* were available; the first one, given by supplier, the second one, calculated by means of these random variables, i.e. through the formula: $granulometry = 0.668 \cdot 220 + 0.332 \cdot 450$. The comparison between two values is here below reported:

$$granulometry = [296.36] \leftrightarrow (294.51)$$

The graph of figure 3 summarizes all the test results, considering that 100 multiply the data points of the first 6 comparisons and that *granulometry* values are divided by 1000; these operations have been performed, to allow the simple picture of the figure 3. A similar situation was found in relation to the other 5 tested lots; hence, relation (3), shown below, was assumed as a satisfactory model for particle weight density.

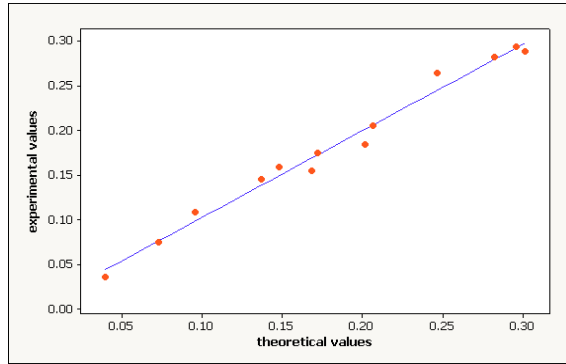


Fig. 3 – Comparison between experimental values and model prediction

$$f(x) = \frac{P_1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{1-P_1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} \quad (3)$$

2. Particle number probability density

Indicating by γ the density of material under study and assuming as spherical the particle shape, the probability density function of particle number, say $\nu(x)$, has been obtained by means of the following equation (4).

$$f(x)dx = \nu(x) \frac{4\pi x^3}{24} \gamma dx \quad (4)$$

i.e.

$$\nu(x) = \frac{6}{\gamma\pi} \frac{f(x)}{x^3} \quad (5)$$

Putting $[f(x)dx]=[mg]$ and $[x]=[micron]$, it resulted that, for the material under study, the relation $\frac{6}{\gamma\pi} = 2.03 \cdot 10^9$ is true. The graphs of functions $f(x)$ and $v(x)$ are reported in figure 4. $v(x)$, in particular, is graphed, starting from $x = 100$, because its numerical values, for low x values increase very rapidly.

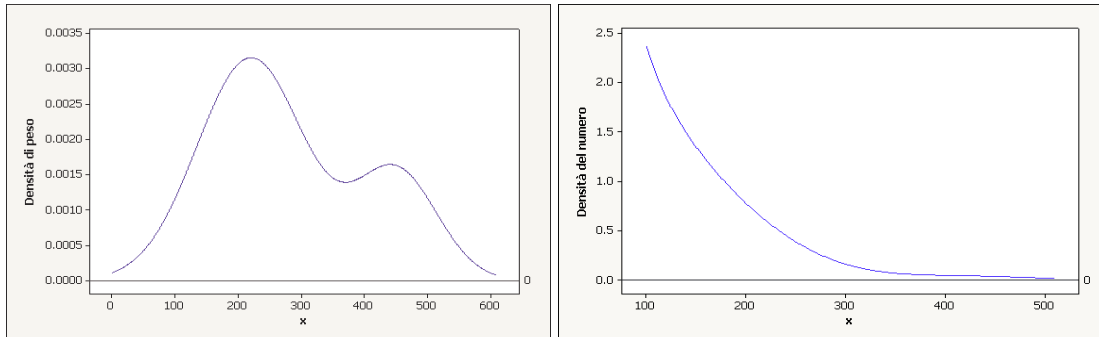


Fig. 4 – Probability density functions of particle weight and number.

3. Discussion and concluding remarks

Data analysis results, performed on 6 lots of powder, spread on a period of 3 years, show a consistent behavior. Powder weight versus size, follows a bimodal distribution; mixture components are pretty constant as well as their standard deviations. Then, difference among lots seems to be connected only to mean variations. Also on the basis of additional analyses, out of the scope of this note, it seems useful identifying powder quality, by means of particle number distribution. In fact, quality and manufacturability of a finished product depend, after all, on the number and size of particles. A better parameter evaluation needs refining the current test method, by introducing new weight measurements in the area of low size particle.

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